AN INTRODUCTION TO RISK AND RETURN

CONCEPTS AND EVIDENCE

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Today, most students of financial management would agree that the treatment of risk is the main element in financial decision making. Key current questions involve how risk should be measured, and how the required return associated with a given risk level is determined. A large body of literature has developed in an attempt to answer these questions.

However, risk did not always have such a prominent place. Prior to 1952 the risk element was usually either assumed away or treated qualitatively in the financial literature. In 1952 an event occurred which was to revolutionize the theory of financial management. In a path-breaking article, an economist by the name of Harry Markowitz [17] suggested a powerful yet simple approach for dealing with risk. In the two decades since, the modern theory of portfolio management has evolved.

Portfolio theory deals with the measurement of risk, and the relationship between risk and return. It is concerned with the implications for security prices of the portfolio decisions made by investors. If, for example, all investors select stocks to maximize expected portfolio return for individually acceptable levels of investment risk, what relationship would result between required returns and risk?

One answer to this question has been developed by Professors Lintner [14, 15] and Sharpe [22], called the Capital Asset Pricing Model. Once such a normative relationship between risk and return is obtained, it has an obvious application as a benchmark for evaluating the performance of managed portfolios.

The purpose of this paper is to present a nontechnical introduction to modern portfolio theory. Our hope is to provide a wide class of readers with an understanding of the foundations upon which risk measures such as "beta", for example, are based. We will present the main elements of the theory along with the results of some of the more important empirical tests. We are not attempting to present an exhaustive survey of the theoretical and empirical literature.

The paper is organized as follows. Section 1 develops measures of investment return which are used in the study. Section 2 introduces the concept of portfolio risk. We will suggest, as did H. Harkowitz in 1952, that the standard deviation of portfolio returns be used as a measure of total portfolio risk. Section 3 deals with the impact of diversification on portfolio risk. The concepts of systematic and unsystematic risk are introduced here. Section 4 deals with the contribution of individual securities to portfolio risk. The nondiversifiable or systematic risk of a portfolio is shown to be a weighted average of the systematic risk of its component securities. Section 5 discusses procedures for measuring the systematic risk or "beta" factors for securities and portfolios. Section 6 presents an intuitive justification of the capital asset pricing model. This model provides a normative relationship between security risk and expected return. Section 7 presents a review of empirical tests of the model. The purpose of these tests is to see how well the

model explains the relationship between risk and return that exists in the securities market. Finally, Section 8 discusses how we can use the capital asset pricing model to measure the performance of institutional investors.

1. INVESTMENT RETURN

Measuring historical rates of return is a relatively straight-forward matter. The return on our investor's portfolio during some interval is equal to the capital gains plus any distributions received on the portfolio. It is important that distributions, such as dividends, be included, else the measure of return to the investor is deficient. The return on the investor's portfolio, designated R_p, is given by

$$R_p = \frac{D_p + \Delta V_p}{V_p} \tag{1}$$

where

D_p = dividends received

ΔV_p = change in portfolio value during the interval (Capital Gains)

V_p = market value of the portfolio at the beginning of the period

The formula assumes no capital inflows during the measurement period. Otherwise the calculation would have to be modified to reflect the increased investment base. Further, the calculation assumes that any distributions occur at the end of the period, or that distributions are held in the form of cash until period end. If the distributions were invested prior to the end of the interval, the calculation would have to be modified to consider gains or losses on the amount reinvested.

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Thus, given the beginning and ending portfolio values and distributions received, we can measure the investor's return using Equation (1). For example, if the investor's portfolio had a market value of $100 at the beginning of June, produced $10 of dividends, and had an end-of-month value of $95, the return for the month would be 0.05 or 5%.

To measure the average return over a series of measurement intervals, two calculations are commonly used: the "arithmetic average" and the "geometric average" returns. We will describe each below. To illustrate the calculations, consider a portfolio with successive annual returns of -0.084, 0.040, and 0.143. Designate these returns as R_1, R_2, and R_3.

The arithmetic return measures the average portfolio return realized during successive 1-year periods. It is simply any unweighted average of the three annual returns; that is, $(R_1 + R_2 + R_3)/3$. The value for the portfolio is 3.3 percent per year.[2]

The geometric average measures the compounded rate of growth of the portfolio over the 3-year period. The average is obtained by taking a "geometric" average of the three annual returns; that is, $\left\{[(1 + R_1)(1 + R_2)(1 + R_3)]^{1/3} - 1.0\right\}$. The resulting growth rate for the portfolio is 2.9% per annum compounded annually, for a total 3-year return of 8.9%.[3]

The geometric average measures the true rate of return while the arithmetic average is simply an average of successive period returns. The distinction can perhaps be made clear by an example. Consider an asset which is purchased for $100 at the beginning of year 1. Suppose the

assets price rises to $200 at the end of the first year and then falls back to $100 by the end of the second year. The arithmetic average rate of return is the average of +100% and -50%, or +25%. But an asset purchased for $100 and having a value of $100 two years later did not earn 25%; it clearly earned a zero return. The arithmetic average of successive one-period returns is obviously not equal to the true rate of return. The true rate of return is given by the geometric mean return defined above; that is, $[(2.0)(0.5)]^{1/2} - 1.0 = 0.$ [4]

In the remainder of the paper, we will refer to both types of averages.

2. PORTFOLIO RISK

The definition of investment risk leads us into much less well explored territory. Not everyone agrees on how to define risk, let alone measure it. Nevertheless, there are some attributes of risk which are reasonably well accepted.

If an investor holds a portfolio of treasury bonds, he faces no uncertainty about monetary outcome. The value of the portfolio at maturity of the notes will be identical with the predicted value. The investor has borne no risk. However, if he has a portfolio composed of common stocks, it will be impossible to exactly predict the value of the portfolio as of any future date. The best he can do is to make a best guess or most likely estimate, qualified by statements about the range and likelihood of other values. In this case, the investor has borne risk.

A measure of risk is the extent to which the <u>future</u> portfolio values are likely to diverge from the expected or predicted value. More specifically, risk for most investors is related to the chance that future portfolio values will be less than expected. Thus, if the investor's portfolio has a current value of $100,000, at an expected value of $110,000 at the end of the next year, he will be concerned about the probability of achieving values less than $110,000.

Before proceeding to the quantification of risk, it is convenient to shift our attention from the terminal value of the portfolio to the portfolio rate of return, R_p. Since the increase in portfolio value is

directly related to R_p, this transformation results in no substantive difference.[5] However, it is convenient for later analysis.

A particularly useful way to quantify the uncertainty about the portfolio return is to specify the probability associated with each of the possible future returns. Assume, for example, that an investor has identified five possible outcomes for his portfolio return during the next year. Associated with each return is a subjectively determined probability, or relative chance of occurrence. The five possible outcomes are:

PossiblelReturn	Subjective Probability
50%	0.1
30%	0.2
10%	0.4
-10%	0.2
-30%	0.1
	1.00

Note that the probabilities sum to 1 so that the actual portfolio return is confined to take one of the five possible values. Given this probability distribution, we can measure the expected return and risk for the portfolio.

The expected return is simply the weighted average of possible outcomes, where the weights are the relative chances of occurrence. The expected return on the portfolio is 10%, given by

$$E(R_p) = \sum_{j=1}^{5} P_j R_j$$

$$= 0.1(50.0) + 0.2(30.0) + 0.4(10.0)$$

$$+ 0.2(-10.0) + 0.2(-30.0)$$

<div align="right">(2)</div>

where the R_j's are the possible returns and the P_j's the associated probabilities. (The expected terminal market value of the portfolio is equal to $M_0(1 + .10)$, where M_0 is the initial value.)

If risk is defined as the chance of loss or achieving returns less than expected, it would seem to be logical to measure risk by the dispersion of the possible returns below the expected value. However, risk measures based on below-the-mean variability are difficult to work with, and furthermore are unnecessary as long as the distribution of future return is reasonably symmetric about its expected values.[6] Figure 1 shows three probability distributions: the first symmetric, the second skewed to the left, and the third skewed to the right. The symmetrical distribution has no skewness. The dispersion of returns on one side of the expected return is a mirror image of the dispersion on the other side of the expected return.

Empirical studies of realized rates of return on diversified port-volios show that skewness is not a significant problem.[7] If the shapes of historical distributions are indicative of the shapes of future distributions, then it makes little difference whether we measure variability of returns on one or both sides of the expected return. Measures of the total variability of return will be twice as large as measures of the portfolio's variability below the expected return if its probability distribution is

symmetric. Thus, if total variability is used as a risk surrogate, the risk rankings for a group of portfolios will be the same as when variability below the expected return is used. It is for this reason that total variability of returns has been so widely used as a surrogate for risk.

It now remains to develop a specific measure of total variability of returns. The measures which are most commonly used are the variance and standard deviation of returns. Measuring risk by standard deviation and variance is equivalent to defining risk as total variability of returns about the expected return, or simply, variability of returns.

The variance of return is a weighted sum of the deviations from the expected return. The variance, designated σ_p^2, for the portfolio in the previous example is given by

$$\sigma_p^2 = \sum_{j=}^{5} P_j \left(R_j - E(R_p) \right)^2$$

$$= 0.1(50.0 - 10.0)^2 + 0.2(30.0 - 10.0)^2$$

$$+ 0.4(10.0 - 10.0)^2 + 0.2(-10.0 - 10.0)^2$$

$$+ 0.1(-30.0 - 10.0)^2$$

$$= 484 \text{ percent squared} \tag{3}$$

The standard deviation is defined as the square root of the variance. It is equal to 22%. The larger the variance or standard deviation, the greater the possible dispersion of future realized values around the expected value,

and the larger the investor's uncertainty. As a rule of thumb, it is often suggested that two-thirds of the possible returns on a portfolio will be within one standard deviation of return either side of the expected value; ninety-five percent will lie with plus or minus two standard deviations of the expected return.

Figure 2 shows the historical return distributions for a diversified portfolio. The portfolio is composed of approximately 100 securities, with each security having equal weight. The month-by-month returns cover the period from January 1945 to June 1970. Note that the distribution is approximately symmetric, but not exactly. The arithmetic average return for the 306-month period is 0.91% per month. The standard deviation about this average is 4.45% per month.

Figure 3 gives the same data for a single security, National Department Stores. The arithmetic average return is 0.81% per month over the 306-month period. The most interesting aspect, however, is the standard deviation of month-by-month returns -- 9.02% per month, more than double that for the diversified portfolio. This result will be discussed further in the next section.

Thus far our discussion of portfolio risk has been confined to a single-period investment horizon such as the next year. That is, the portfolio is held unchanged and evaluated at the end of the year. An obvious question relates to the effect of holding the portfolio for several periods, such as the next 20 years: will the 1-year risks tend to cancel out over time? Given the random walk nature of security prices, the answer to this question is no. If the risk level (standard deviation) is maintained during each year, the portfolio risk for longer horizons will

increase with the horizon length. The standard deviation of possible

terminal portfolio values after N years is equal to $\sqrt{N}$ times the

standard deviation after 1 year.[9/] Thus, the investor cannot rely on the

"long run" to reduce his risk of loss.

A final remark before leaving portfolio risk measures. We have

implicitly assumed that investors are risk averse, i.e., they seek to

minimize risk for a given level of return. This assumption appears to

be valid for most investors in most situations. The entire theory of

portfolio selection and capital asset pricing is based on the belief that

investors on the average are risk averse.

3. DIVERSIFICATION

When the distribution of historical returns for the 100-stock portfolio (Figure 2) is compared with the distribution for National Department Stores (Figure 3), a curious relationship is discovered. While the standard deviation of returns for the security is doublt that of the portfolio, its average return is less. Is the market so imperfect that over a long period of time (25 years) it rewarded substantially higher risk with lower average return?

No so. Much of the total risk (standard deviation of return) of National Department Stores is diversifiable. That is, when combined with other securities, a portion of the variation of its returns is smoothed or cancelled by complementary variation in the other securities. Since much of the total risk could be eliminated simply by holding the stock in a portfolio, there was no economic requirement for the return earned to be in line with the total risk. Instead, we should expect realized returns to be related to that portion of security risk which cannot be eliminated by portfolio combination (more on risk-return relationships later). The same portfolio diversification effect accounts for the low standard deviation of return for the 100-stock portfolio. In fact, the portfolio standard deviation is less than that of the typical security in the portfolio. Much of the total risk of the component securities has been eliminated by diversification.

Diversification results from combining securities which have less than perfect correlation (dependence) among their returns in order to reduce portfolio risk without sacrificing portfolio return. In general, the

lower the correlation among security returns, the greater the impact of diversification. This is true regardless of how risky the securities of the portfolio when considered in isolation.

Ideally, if we could find sufficient securities with uncorrelated returns, we could completely eliminate portfolio risk. However, this situation is not typical of real securities markets in which securities' returns are positively correlated to a considerable degree. Thus, while portfolio risk can be substantially reduced by diversification, it cannot be entirely eliminated. This can be demonstrated very clearly by measuring the standard deviations of randomly selected portfolios containing various numbers of securities.

In a study of the impact of portfolio diversification on risk, Wagner and Lau [24] divided a sample of 200 NYSE stocks into six subgroups based on the Standard and Poors Stock Quality Ratings as of June 1960. The highest quality ratings (A+) formed the first group, the second highest ratings (A) the next group, and so on. Randomly selected portfolios were then formed from each of the subgroups, containing from 1 to 20 securities. The month-by-month portfolio returns for the 10-year period through May 1970 were then computed for each portfolio (portfolio composition remaining unchanged). The exercise was repeated ten times to reduce the dependence on single samples. The values for the ten trials were then averaged.

Table 1 shows the average return and standard deviation for portfolios from the first subgroup (A+ quality stocks). The average return is unrelated to the number of issues in the portfolio. On the other hand, the standard deviation of return declines as the number of holdings increases. On the average, approximately 40% of the single security risk is eliminated

by forming randomly selected portfolios of 20 stocks. However, it is also evident that additional diversification yields rapidly diminishing reduction in risk. The improvement is slight when the number of securities held is increased beyond, say, 10. Figure 4 shows the results for all six quality groups. The figure shows the rapid decline in total portfolio risk as the portfolios are expanded from 1 to 10 stocks.

Returning to Table 1, we note from the second last column in the table that the return on a diversified portfolio "follows the market" very closely The degree of association is measured by the correlation (R) of each portfolio with an unweighted index of all NYSE stocks. The 20-security portfolio has a correlation of 0.89 with the market (perfect positive correlation results in a correlation of 1.0).[9/] The implication is that the risk remaining in the 20-stock portfolio is predominantly a reflection of uncertainty about the performance of the stock market in general. Figure 5 shows the results for the six quality groups. Correlation in Figure 5 is given by the correlation coefficient squared, designated R^2 (possible values range from 0 to 1.0).

The R-squared coefficient has a useful interpretation. It measures the proportion of variation in portfolio return which is attributable to variation in market returns. The remaining variation is risk which is unique to the portfolio and, as we saw in Figure 4, can be eliminated by proper diversification of the portfolio. Thus, R^2 measures the degree of portfolio diversification. A poorly diversified portfolio will have a small R-squared (0.30 - 0.40). A well diversified portfolio will have a much higher R squared (0.85 - 0.95). A perfectly diversified portfolio will have

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an R-squared of 1.0; that is, all of the portfolio risk is a reflection of market risk. Figure 5 shows the rapid gain in diversification as the portfolio is expanded from one security to two securities and up to ten securities. Beyond ten securities the gains tend to be smaller. Note that the highest quality A+ issues tend to be less efficient at achieving diversification for a given number of issues. Apparently the companies which comprise this group are more homogeneous than the companies grouped under the other quality codes.

The results show that some risks can be eliminated via diversification, others cannot. Thus we are led to the distinction between a portfolio's unsystematic risk, which can be eliminated by diversification, and its systematic risk which cannot. The situation is depicted in Figure 6. The figure shows total portfolio risk declining with increasing numbers of holdings. The total risk of the portfolio is made up to two parts: systematic or nondiversifiable risk and unsystematic risk. Unsystematic risk is gradually eliminated with increased numbers of holdings until portfolio risk is entirely systematic, i.e., market related. The systematic risk is due to the fact that the return on nearly every security depends to some degree on the overall performance of the stock market. Investors are thus exposed to "market uncertainty" no matter how many stocks they hold. Consequently, the return on diversified portfolios is highly correlated with the market.

4. THE RISK OF INDIVIDUAL SECURITIES

Let's summarize the message of the previous section. Portfolio risk can be divided into two parts: systematic and unsystematic risk. Unsystematic risk can be eliminated by portfolio diversification, systematic risk cannot. When unsystematic risk has been completely eliminated, portfolio return is perfectly correlated with the market. Portfolio risk is then merely a reflection of the uncertainty about the performance of the market.

The systematic risk of a portfolio is made up from the systematic risks of its component securities. The systematic risk of an individual security is that portion of its total risk (standard deviation of return) which cannot be eliminated by placing it in a well-diversified portfolio. We now need a way of quantifying the systematic risk of a security and evaluating the systematic risk of a portfolio from its component securities.

The nature of security risk can be better understood by dividing security return into two parts: one dependent (i.e., perfectly correlated), and a second independent (i.e., uncorrelated) of market return. The first component of return is usually referred to as "systematic", the second as "unsystematic" return. Thus,

$$\text{Security Return} = \text{Systematic Return}$$
$$+ \text{Unsystematic Return} \tag{4}$$

Since the systematic return is perfectly correlated with the market return, it can be expressed as a factor, designated beta (β), times the market return, R_m. The "beta" factor is a "market sensitivity index", indicating how sensitive the security return is to changes in the market level. The unsystematic return, which is independent of market returns, is usually represented by a factor epsilon (ϵ). Thus, the return on a security, R, may be expressed as

$$R = \beta R_m + \epsilon \tag{5}$$

For example, if a security had a β factor of 2.0 (e.g., an airline stock), then a 10% market return would generate a systematic return for the stock of 20%. The security return for the period would be the 20% plus the unsystematic component. The unsystematic return depends on factors unique to the company, such as labor difficulties, higher-than-expected sales, etc.

The security returns model given by Equation (5) is usually written in a way such that the average value of the residual term, ϵ, is zero. This is accomplished by adding a factor, alpha (α), to the model to represent the average value of the unsystematic returns over time. That is,

$$R = \alpha + \beta R_m + \epsilon \tag{6}$$

where the average ϵ over time is equal to zero.

The model for security returns given by Equation (6) is usually referred to as the "market model". Graphically, the model can be depicted as a line fitted to a plot of security returns against rates of

return on the market index. This is shown in Figure 7 for a hypothetical security.

The beta factor can be thought of as the slope of the line. It gives the expected increase in security return for a 1% increase in market return. In Figure 7, the security has a beta of 1.0. Thus, a 10% market return will result, on the average, in a 10% gain in security price. The market weighted average beta for all stocks is 1.0 by definition.

The alpha factor is represented by the intercept of the line on the vertical security return axis. It is equal to the average value overtime of the unsystematic returns on the stock. For most stocks, the alpha factor tends to be small and unstable.

Using the definition of security return given by the market model, the specification of systematic and unsystematic risk is straightforward -- they are simply the standard deviations of the two return components.$\underline{10/}$

The systematic risk of a security is equal to β times the standard deviation of the market return.

$$\text{Systematic Risk} \quad = \quad \beta \sigma_m \qquad\qquad (7)$$

The unsystematic risk equals the standard deviation of the residual return factor ϵ.

$$\text{Unsystematic Risk} \quad = \quad \sigma_\epsilon \qquad\qquad (8)$$

Given measures of security systematic risk, we can now compute the systematic risk of a portfolio. It is equal to the beta factor for the portfolio, β_p, times the risk of the market index, σ_m.

$$\text{Portfolio Systematic Risk} \quad = \quad \beta_p \sigma_m \qquad\qquad (9)$$

The portfolio beta factor in turn can be shown to be simply an average of the individual security betas, weighted by the proportion of each security in the portfolio, or

$$\beta_p = \sum_{j=1}^{N} X_j \, \beta_j \qquad (10)$$

where

X_j = the proportion of portfolio market value represented by security j

N = the number of securities

Thus, the systematic risk of the portfolio is simply a weighted average of the systematic risk of the individual securities. If the portfolio is composed of an equal dollar investment in each stock (as was the case for the 100-security portfolio of Figure 2), the β_p is simply an unweighted average of the component security betas.

The unsystematic risk of the portfolio is also a function of the unsystematic security risks, but the form is more complete.[11/] With increasing diversification, this risk can be eliminated

With these results for portfolio risk, it is useful to return to Figure 4. The figure shows the decline in portfolio risk with increasing diversification for each of the six quality groups. However, the portfolio standard deviations for each of the six groups are approaching different limits. We should expect these limits to differ because the average risks (β) of the groups differ.

Table 2 shows a comparison of the standard deviations for the 20-stock portfolios with the predicted lower limits based on average security systematic risks. The lower limit is equal to the average beta

for the quality group $(\overline{\beta})$ times the standard deviation of the market return (σ_m). The standard deviations in all cases are close to the predicted values. These results support the contention that portfolio systematic risk equals the average systematic risks of the component securities.

Before moving on, let's summarize the results of this section. First, as seen from Figure 4, roughly 40 to 50% of total security risk can be eliminated by diversification. Second, the remaining systematic risk is equal to the security β times market risk. Thirdly, portfolio systematic risk is a weighted average of security systematic risks.

The implications of these results are substantial. First, we would expect realized rates of return over substantial periods of time to be related to the systematic as opposed to total risk of securities. Since the unsystematic risk is relatively easily eliminated, we should not expect the market to pay a "risk premium" for bearing it.

Second, since security systematic risk is equal to the security beta times σ_m (which is common to all securities), beta can be considered as a relative risk measure. The β gives the systematic risk of a security (or portfolio) relative to the risk of the market index. It is more convenient to speak of systematic risk in terms of the beta factor, rather than beta times σ_m.

5. MEASUREMENT OF SECURITY
AND PORTFOLIO BETA VALUES

The basic data for estimating betas are past rates of return earned over a series of relatively short intervals -- usually days, weeks, or months. For example, in Tables 3 and 4 we present calculations based on month-by-month rates of returns for the periods January 1945 to June 1970 (security betas) and January 1960 to December 1971 (mutual fund betas). The returns are calculated in the manner described in Section 1 (see Equation (1)).

It is customary to convert the observed rates of returns to "risk premiums". Risk premiums are obtained by subtracting the rates of return that could have been achieved by investing in short-maturity risk-free assets, such as treasury bills or prime commercial paper. This removes a source of "noise" from the data. The noise stems from the fact that observed returns may be higher in some years simply because risk-free rates of interest are higher. Thus, an observed rate of return of 8% might be regarded as satisfactory if it occurred in 1960, but as a relatively low rate of return when interest rates were at all-time highes in 1969. Rates of return expressed as risk premiums will be denoted by small r's.[12]

Beta for a security is calculated by fitting a straight line to the plot of observed returns r versus observed returns on the market, denoted by r_m. The equation of the fitted line is

$$r = \hat{\alpha} + \hat{\beta} r_m + \hat{\epsilon} \qquad (11)$$

where $\hat{\alpha}$ is the intercept of the fitted line and $\hat{\beta}$ represents the stock's systematic risk. The $\hat{\epsilon}$ term represents variation about the line resulting from the unsystematic component of return. We have put hats ($\wedge$) over the α, β and ϵ terms to indicate that these are estimated values. It is important to remember that these estimated values may differ substantially from the true values because of statistical measurement difficulties. However, the extent of possible error can be measured, and we can indicate a range within which the true value is almost certain to lie.

Figure 8 shows a rate-of-return plot and fitted line for National Department Stores. The market is represented by a market weighted index of all NYSE securities. The plot is based on monthly data during the period January 1945 to June 1970.

The estimated beta is 1.26 indicating above-average systematic risk. The estimated alpha is -0.05% per month, indicating that the non-market-related component of return averaged -0.60% per year over the 25-year period. The correlation coefficient is 0.52; thus, 27% of the variance of security returns resulted from market movements. The remainder was due to factors unique to the company.

Our interpretation of the estimated alpha and beta values must be conditioned by the degree of possible statistical measurement error. The measurement error is estimated by "standard error" coefficients associated with alpha and beta.

For example, the standard error of beta is 0.12. Thus, the probability is about 66% that the true beta will lie between 1.26 $\pm$ 0.12, and about 95% between 1.26 $\pm$ 0.24 (i.e., plus or minus two times the standard error). Thus, we can say with high confidence that National

Department Stores has above-average systematic risk (the average stock has beta = 1.0).

The standard error for alpha is 0.45, which is large compared with the estimated value of -0.05. Thus, we cannot conclude that the true alpha is different from zero, since zero lies well within the range of estimated alpha plus or minus one standard error (i.e., -0.05 ± 0.45).

The process of line fitting used to estimate the coefficients is called "Regression Analysis". Table 4 presents the same type of regression results for a random collection of 30 NYSE stocks.[13] The table contains the following items. Column (1) gives the number of monthly observations, columns (2) and (3) the estimated alpha ($\hat{\alpha}$) and its standard error, columns (4) and (5) the estimated beta ($\hat{\beta}$) and its standard error, column (6) the unsystematic risk $\hat{\sigma}_\epsilon$ (designated SE·R in table), column (7) the R-squared in percentage terms, columns (8) and (9) the arithmetic average of monthly riak premiums ($\bar{r}$) and the standard deviation, column (10) the geometric mean risk premium ($\bar{g}$). The results are ranked in terms of descending values of estimated beta. The table includes summary results for the NYSE market index and the prime commercial paper "risk-free rate".[14] The last two rows of the table give average values and standard deviations for the sample. The average beta, for example, is 1.05, slightly higher than the average of all NYSE stocks. The average alpha is 0.13% per month, indicating a slightly positive average unsystematic return.

The beta value for a portfolio can be estimated in two ways. One method is to computer the beta of all portfolio holdings and weight the results by portfolio representation. This method has the disadvantage of requiring beta calculations for each individual portfolio asset. The second

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method is to use the same computation procedures used for stocks, but applied to the portfolio returns. In this way we can obtain estimates of portfolio betas without explicit consideration of the portfolio securities. We have used this approach to compute portfolio and mutual fund beta values.

Figure 9 shows the plot of the monthly returns on the 100-stock portfolio against the NYSE index for the same 1945 - 1970 period. As in the case of National Department Stores, the best-fit line has been put through the points using regression analysis. The slope of the line ($\hat{\beta}$) is equal to 1.10, with a standard error of 0.03. Note the substantial reduction in the standard error term compared to the security examples. The estimated alpha is 0.14, with a standard error of 0.10. Again, we cannot conclude that the true alpha is different from zero. Note that the points group much closer to the line than in the National Department Store plot. This results, of course, from the fact that much of the unsystematic risk which causes the points to be scattered around the regression line in Figure 8 has been eliminated. The reduction is evidenced by the R-squared measure of 0.87 (versus 0.27 for National Department Stores). Thus, the market explains more than three times as much of the return variation of the portfolio than for the stock.

Table 5 gives regression results for a sample of 49 mutual funds. The calculations are based on monthly returns for the period January 1960 to December 1971. The market is represented by the Standard & Poor's 500 Stock Index. Average values and standard deviations for the 49 funds in the sample are shown in the last two rows of the table. The average beta value for the group is 0.92 indicating, on the average, the funds were

less risky than the market index. Note the relatively low beta values of the balanced and bond funds, in particular, the keystone B1, B2, and B4 bond funds. This result is due to the low correlation between bond and stock returns.

Up to this point we have shown that it is a relatively easy matter to estimate beta values for stocks, portfolios, and mutual funds. Now, if the beta values are to be useful for investment decision making, they must be predictable. That is, beta values based on historical data should provide considerable information about future beta values if past measures are to be useful. The question can be asked at three levels. How predictable are the betas estimated for stocks, portfolios of stocks, and mutual funds? Fortunately, we have empirical evidence at each level.

Robert A. Levy [13] has conducted tests of the short-run predictability (also referred to as stationarity) of beta coefficients for securities and unmanaged portfolios of securities. Levy's results are based on weekly returns for 500 NYSE stocks for the period December 30, 1960 through December 18, 1970 (520 weeks). Betas were developed for each security for ten non-overlapping 52-week periods. To measure stationarity, Levy correlated the 500 security betas from each 52-week period (the historical historical betas) with the 52-week betas in the following period (the future betas). Thus, nine correlation studies were performed for the ten periods.

To compare the stationarity of security and portfolio betas, Levy constructed portfolios of 5, 10, 25, and 50 securities and repeated the same correlation analysis for the historical portfolio betas and future beta

values for the same portfolios in the subsequent period. The portfolios were constructed by ranking security betas in each period and partitioning the list into portfolios containing 5, 10, 25, and 50 securities. Each portfolio contained an equal investment in each security.

The results of Levy s 52-week correlation studies are presented in Table 5. The average values of the correlation coefficients from the nine trials were 0.486, 0.769, 0.853, 0.939, and 0.972 for portfolios of 1, 5, 10, 25, and 50 stocks, respectively. Correspondingly, the average percentages of the variation in future betas explained by the historical betas are 23.6, 59.1, 72.8, 88.2, and 94.5.

The results show the beta coefficients to be very predictable for large portfolios, and of progressively declining predictability for smaller portfolios and individual securities. These conclusions are not affected by changes in market performance. Of the nine correlation studies, five covered forecast periods during which the market performance was the reverse of the preceding period (61-62, 62-63, 65-66, 66-67, and 68-69). Notably, the betas were approximately as predictable over these five reversal periods as over the remaining four intervals.[15]

The question of the stability of mutual fund beta values is more complicated. Even if, as seen above, the betas of large unmanaged portfolios are very predictable, there is no a priori need for mutual fund betas to be comparatively stable. Indeed, mutual fund portfolios are managed, and as such, the betas may change substantially over time by design. For example, a portfolio manager would tend to reduce the

risk exposure of his fund prior to an expected market decline and raise it prior to an expected market upswing. However, the range of possible values for beta will tend to be restricted, at least in the longer run, by the fund's investment objective. Thus, while we do not expect the same standard of predictability as for large unmanaged portfolios, it is of interest to examine the extent to which fund betas are predictable.

Pogue and Conway [20] have conducted preliminary tests for a sample of 90 mutual funds. The beta values for the period January 1969 through May 1970 were correlated with values from the subsequent period from June 1970 through October 1971. To test the sensitivity of the results to changes in the return measurement interval, the betas for each sub-period were measured for daily, weekly, and monthly returns. The betas were thus based on very different numbers of observations, namely 357, 74, and 17, respectively. The resulting correlation coefficients were 0.915, 0.895, and 0.703 for daily, weekly, and monthly betas, respectively. Correspondingly, the average percentages of variation in second-period betas explained by first-period values are 84, 81, and 49, respectively. The results support the contention that historical betas contain useful information about future values. However, the degree of predictability depends on the extent to which measurement errors have been eliminated from beta estimates. In the Pogue-Conway study, the shift from monthly to daily returns reduced the average standard error of the estimated beta values from 0.11 to 0.03, a 75% reduction. The more accurate daily estimates resulted in a much higher degree of beta predictability, the correlation between sub-period betas increasing from 0.703 to 0.915. [16]

Figure 10 shows a plot of the Pogue-Conway first-period versus second-period betas based on daily returns. The figure illustrates the high degree of correlation between first- and second-period betas.

In summary, we can conclude that estimated security betas are not highly predictable. Levy's tests indicated that an average on 24% of the variation in second-period betas is explained by historical values. The betas of his portfolios, however, were much more predictable, the degree of predictability increasing with portfolio diversification. The results of the Pogue and Conway study (among others, see footnote 16) show that fund betas are not as stable as those for unmanaged portfolios. On the average, two-thirds to three-quarters of the variation in fund betas can be explained by historical values.

Further, it should be remembered that a significant portion of the measured changes in estimated beta values may not be due to changes in the true values, but rather the result of measurement errors. This observation is particularly applicable to individual security betas where the standard errors tend to be large.

6. THE RELATIONSHIP BETWEEN
EXPECTED RETURN AND RISK

We have now developed two measures of risk and described how they can be measured from historical data. One is a measure of total risk (standard deviation), the other a relative index of systematic or nondiversifiable risk (beta). We have stated our belief that the beta measure is more relevant for the pricing of securities. Returns expected by investors should logically be related to systematic as opposed to total risk. Securities with higher systematic risk should have higher expected returns.[17/]

The question of interest now is the form of the relationship between risk and return. In this section we describe a relationship called the "Capital Asset Pricing Model" (CAPM), which is based on elementary logic and simple economic principles. The basic postulate underlying the model is that assets with the same risk should have the same expected rate of return. That is, the prices of assets in the capital markets should adjust until equivalent risk assets have identical expected returns. At this point, we say that the market is in an "equilibrium" condition.

To see the implications of this postulate, consider an investor who holds a portfolio[18/] with the same risk as the market portfolio (beta equal to 1.0). What return should he expect? Logically, he should expect the same return as that of the market portfolio.

Consider another investor who holds a riskless portfolio (beta equal to zero). The investor in this case should expect to earn the rate of return on riskless assets such as treasury bills. By taking no risk, he earns the riskless rate of return.

Now consider the case of an investor who holds a mixture of these two portfolios. Assume he invests a proportion X of his money in the risky portfolio and (1 - X) in the riskless portfolio. What risk does he bear and what return should he expect? The risk of the composite portfolio is easily computed. Recall that the beta of a portfolio is simply a weighted average of the component security betas, where the weights are the portfolio proportions. Thus, the portfolio beta, β_p, is a weighted average of the market and risk-free rate betas, that is, an average of zero and one. Thus

$$\beta_p = (1 - X) \cdot 0 + X \cdot 1$$

$$= X \tag{12}$$

Thus, β_p is equal to the fraction of his money invested in the risky portfolio. If 100% or less of the investor's funds are invested in the risky portfolio, his portfolio beta will be between zero and 1.0. If he borrows at the risk-free rate and invests the proceeds in the risky portfolio, his portfolio beta will be greater than 1.0.

The expected return of the composite portfolio is also a weighted average of the expected returns on the two-component portfolios; that is,

$$E(R_p) = (1 - X) \cdot R_F + X \cdot E(R_m) \tag{13}$$

where $E(R_p)$, $E(R_m)$, and R_F are the expected returns on the portfolio, the market index, and the risk-free rate. Now, from Equation (12) we know that X is equal to β_p. Substituting into Equation (13), we have

$$E(R_p) = (1 - \beta_p) \cdot R_F + \beta_p \cdot E(R_m)$$

$$= R_F + \beta_p \cdot \left(E(R_m) - R_F \right) \qquad (14)$$

Equation (14) is the Capital Asset Pricing Model (CAPM). It is an extremely important theoretical result. It says that the expected return on a portfolio should exceed the riskless rate of return by an amount which is proportional to the portfolio beta. That is, the relationship between return and risk should be linear.

The model is often stated in risk-premium form,

$$E(r_p) = \beta_p \cdot E(r_m) \qquad (15)$$

where $E(r_p)$ and $E(r_m)$ are the expected portfolio and market risk premiums, formed by subtracting the risk-free rate from the rates of return. In this form the model states that the expected risk premium for the investor's portfolio is equal to its beta value times the expected market risk premium.

We can illustrate the model by assuming that the short-term (risk-free) interest rate is 6% and the expected return on the market with a relative risk (beta) of 1.0 is 10%. The expected risk premium for holding the market portfolio is just the difference between the 10% and the short-term interest rate of 6%, or 4%. Investors who hold the market portfolio expect to earn 10%, which is 4% greater than they could earn on a

short-term market instrument for certain. The expected return on securities with different levels of risk should be as follows.

Expected Return for Different Levels of Portfolio Beta

Beta	Expected Return
0.0	6%
0.5	8%
1.0	10%
1.5	12%
2.0	14%

The predictions of the model are inherently sensible. For safe investments ($\beta = 0$), the model predicts that investors would expect to earn the risk-free rate of interest. For a risky investment ($\beta > 0$) investors would expect a rate of return proportional to the market sensitivity (β) of the stock. Thus, stocks with lower-than-average market sensitivities (such as most utilities) would offer expected returns less than the expected market return. Stocks with above-average values of beta (such as most airline securities) would offer expected returns in excess of the market.

In our development of the CAPM we have implicitly made a number of assumptions which are required if the model is to be established on a rigorous basis. These assumptions involve investor behavior and conditions in the capital markets. The following is a set of assumptions which are sufficient to allow a simple derivation of the model.

(a)   The market is composed of risk-averse investors who measure risk in terms of standard deviation of portfolio

return. This assumption provides a basis for the use of beta-type risk measures.

(b) All investors have a common time horizon for investment decision making (e.g., 1 month, 1 year, etc.). This assumption allows us to measure investor expectations over some common interval, thus making comparisons meaningful.

(c) All investors are assumed to have the same expectations about future security returns and risks. Without this assumption, investors would disagree on expected return and risks, resulting in a more complex situation.

(d) Capital markets are perfect in the sense that all assets are completely divisible, there are no transactions costs or differential taxes, and borrowing and lending rates are equal to each other and the same for all investors. Without these conditions, frictional barriers would exist to the equilibrium conditions on which the model is based.

While these assumptions are sufficient to derive the model, it is not clear that all are necessary in their current form. It may well be that several of the assumptions can be substantially relaxed without major change in the form of the model. A good deal of research is currently being conducted toward this end.

While the CAPM is indeed simple and elegant, these qualities do not in themselves make it useful in explaining observed risk-return patterns. We now proceed to the empirical literature on attempts to verify the model.

7. TESTS OF THE CAPITAL ASSET
PRICING MODEL[19/]

The major difficulty in testing is that the Capital Asset Pricing Model is stated in terms of investors' expectations, not in terms of realized returns. Obviously, expectations are not always realized after the fact. From a statistical point of view, this introduces an error term which should be zero on the average, but not necessarily zero for any single stock of single period of time. After the fact, we would expect to observe

$$R_j = R_f + \beta_j(R_m - R_f) + \epsilon_j \qquad (16)$$

where R_j, R_m, and R_f are the realized returns on stock j, the market index, and the riskless asset; and ϵ_j is the residual term.

The term ϵ_j reflects the firm's unsystematic risk -- the risk due to factors unique to the company. Unsystematic risk is eliminated when the stock is included in a well-diversified portfolio.

Thus, if the hypothesis is correct, and we observe returns for many stocks and many periods of time, then ϵ_j ought to be zero on the average, and the observed risk premiums on various stocks ought to be proportional to the stocks' betas.

This hypothesis is illustrated by Figure 11. Each plotted point represents one stock's realized return versus the stock's beta. According to the capital asset pricing model, the line fitted to these points should be (1) linear and (2) upward sloping. Also, (3) it should pass through the vertical axis at the risk-free rate.

The equation of this fitted line is

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \mu_j \qquad \text{(17a)}$$

where $\bar{R}_j$ is the average return realized on stock j, $\hat{\beta}_j$ is its estimated beta, and μ_j is a residual term. The capital asset pricing model predicts that γ_0, the intercept of the fitted line on the return axis, should have the value $\bar{R}_f$, and that the slope, γ_1, should have a value equal to $\bar{R}_M - \bar{R}_f$, where $\bar{R}_M$ and $\bar{R}_f$ are the averages of the market returns and risk-free rates of interest during the period studied.

Expressed in risk premium form, the equation of the fitted line is

$$\bar{r}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \mu_j \qquad \text{(17b)}$$

where $\bar{r}_j$ is the average realized risk premium stock j, that is, $\bar{R}_j - \bar{R}_f$. The only difference is that the predicted value of γ_0 under the capital asset pricing model hypothesis is zero.

Other Measures of Risk

The hypothesis just described is only true if beta is a complete measure of a stock's risk. Various alternative risk measures have been proposed. The most common alternative hypothesis is that expected return is related to the standard deviation of return -- that is, to a stock's total risk, which includes both systematic and unsystematic components.

What is more important in explaining average observed returns on securities, systematic or unsystematic risk? The way to find out is to fit an expanded equation to the data:

$$R_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_2(\hat{SE}_j) + \mu_j \tag{18}$$

Here $\hat{\beta}_j$ is a measure of systematic risk and $\hat{SE}_j$ a measure of unsystematic risk.[20] Of course, if the capital asset pricing model is exactly true, then γ_2 will be zero -- that is, $\hat{SE}_j$ will contribute nothing to the explanation of observed security returns.

Empirical Tests of the Capital Asset Pricing Model

If the capital asset pricing model is right, the empirical tests whould show the following:

1. On the average, and over long periods of time, the securities with high systematic risk should have high rates of return.

2. On the average, there should be a linear relationship between systematic risk and return.

3. Unsystematic risk, as measured by $\hat{SE}_j$, should play no significant role in explaining differences in security returns.

These predictions have been tested in several recent statistical studies. We will review some of the more important of these. Readers wishing to skip the details may proceed to the summary at the end of this section.

We will begin by summarizing results from studies based on individual securities. Then we will turn to portfolio results.

Results for Tests Based on Securities

We will review two studies, one by Professor N. L. Jacob [9], and a second by Professor M. H. Miller and M. S. Scholes [19].

The Jacob Study

This study deals with the 593 New York Stock Exchange stocks for which there is complete data from 1946 to 1965. Regression analyses were performed for the 1946-55 and 1956-65 periods, using both monthly and annual security returns. The relationship of mean security returns and beta values is shown in Table 6. The last two columns of the table give the theoretical values for the coefficients, as predicted by the capital asset pricing model.

The results show a significant positive relationship between realized return and risk during each of the 10-year periods. For example, in 1956-65 there was a 6.7 percent per year increase in average return for a one-unit increase in beta. Although the relationships shown in Table 6 are all positive, they are weaker than predicted by the capital asset pricing model. In each period γ_1 is less than the theoretical value.

The Miller-Scholes Study

The Miller-Scholes research deals with annual returns for 631 stocks during the 1954-63 period. The results of three of their tests are reported in Table 7. The tests are (1) mean return versus beta, (2) mean return versus unsystematic risk, $(S\hat{E}_j)^2$, and (3) mean return versus both beta and unsystematic risk.

The results for the first test show a significant positive relationship between mean return and beta. A one-unit increase in beta is associated with a 7.1 percent increase in mean return.

The results for the second test do not agree with the capital asset pricing model's predictions. That is, high unsystematic risk is apparently associated with higher realized returns. However, Miller and Scholes show that this correlation may be largely spurious (i.e., it may be due to statistical sampling problems). For example, a substantial positive correlation exists between beta and $(\hat{SE}_j)^2$. Thus, even though unsystematic risk may be unimportant to the pricing of securities, it will appear to be significant in tests from which beta has been omitted. This sort of statistical correlation need not imply a causal link between the variables.

Test number (3) includes both beta and $(\hat{SE}_j)^2$ in the regression equation. Both are found to be significantly positively related to mean return. The inclusion of $(\hat{SE}_j)^2$ has somewhat weakened the relationship of return and beta, however. A one-unit increase in beta is now associated with only a 4.2 percent increase in mean return.

The interpretation of these results is again complicated by the strong positive correlation between beta and $(\hat{SE}_j)^2$, and by other sampling problems.[21/] A significant portion of the correlation between mean return and $(\hat{SE}_j)^2$ may well be a spurious result. In any case, the results do show that stocks with high systematic risk tend to have higher rates of return.

Results for Tests Based on Portfolio Returns

The security tests clearly show the significant positive correlation between return and systematic risk. Tests based directly on securities,

however, are not the most efficient method of obtaining estimates of the magnitude of the risk-return tradeoff. Tests based on securities are inefficient for two reasons.

The first problem is well known to economists. It is called "errors in variables bias" and results from the fact that beta, the independent variable in the test, is typically measured with some error. These errors are random in their effect -- that is, some stocks' betas are overestimated and some are underestimated. Nevertheless, when these estimated beta values are used in the test, the measurement errors tend to attenuate the relationship between mean return and risk.

By carefully grouping the securities into portfolios, much of this measurement error problem can be eliminated. The errors in individual stocks' betas cancel out so that the portfolio beta can be measured with much greater precision. This in turn means that tests based on portfolio returns will be more efficient than tests based on security returns.

The second problem relates to the obscuring effect of residual variation. Realized security returns have a large random component, which typically accounts for about 70 percent of the variation of return. (This is the diversifiable or unsystematic risk of the stock.) By grouping securities into portfolios, we can eliminate much of this "noise", and thereby get a much clearer view of the relationship between return and systematic risk.

It should be noted that grouping does not distort the underlying risk-return relationship. The relationship that exists for individual securities is exactly the same for portfolios of securities.

We will review the results from four studies based on portfolios --
two by Professors M. Blume and I. Friend [3] [8], a third by Professors
F. Black, M. Jensen, and M. Scholes [1], and a fourth by E. Fama and
J. MacBeth [6].

Blume and Friend's Study

Professors Blume and Friend have conducted two inter-related
risk-return studies. The first examines the relationship between long-run
rates of return and various risk measures. The second is a direct test of
the capital asset pricing model.

In the first study [8], the authors constructed portfolios of NYSE
common stocks at the beginning of three different holding periods. The
periods began at the ends of 1929, 1948, and 1956. All stocks for which
monthly rate-of-return data could be obtained for at least 4 years
preceding the test period were divided into 10 equal portfolios. The
securities were assigned on the basis of their betas during the preceding
4 years -- the 10 percent of securities with the lowest betas to the first
portfolio, the group with the next lowest betas to the second portfolio,
and so on.

After the start of the test periods, the securities were reassigned
annually. That is, each stock's estimated beta was recomputed at the end
of each successive year, the stocks were ranked again on the basis of
their betas, and new portfolios were formed. This procedure kept the
portfolio betas reasonably stable over time.

The performance of these portfolios is summarized in Table 8.
The table gives the arithmetic mean monthly returns and average beta
values for each of the 10 portfolios and for each test period.

For the 1929-69 period, the results indicate a strong positive association between return and beta. For the 1948-69 period, while higher beta portfolios had higher returns than portfolios with lower betas, there was little difference in return among portfolios with betas greater than 1.0. The 1956-69 period results do not show a clear relationship between beta and return.

On the basis of these and other tests, the authors conclude that NYSE stocks with above-average risk have higher returns than those with below-average risk, but that there is little payoff for assuming additional risk within the group of stocks with above-average betas.

In their second study [3], Blume and Friend used monthly portfolio returns during the 1955-68 period to test the capital asset pricing model. Their tests involved fitting the coefficients of Equation (17a) for three sequential periods: 1955-59, 1960-64, and 1965-68. The authors also added a factor to the regression equation to test for the linearity of the risk return relationship.[22/]

Blume and Friend conclude that "the comparisons as a whole suggest that a linear model is a tenable approximation of the empirical relationship between return and risk for NYSE stocks over the three periods covered.[23/]

The values obtained for γ_0 and γ_1 are not in line with the capital asset pricing models predictions, however. In the first two periods, γ_0 is substantially larger than the theoretical value. In the third period, the reverse situation exists, with γ_0 substantially less than predicted. These results imply that γ_1, the slope of the fitted line, is less than predicted in the first two periods and greater in the third.[24/]

Black, Jensen, and Scholes

This study [1] is a careful attempt to reduce measurement errors that would bias the regression results. For each year from 1931 to 1965, the authors grouped all NYSE stocks into 10 portfolios. The number of securities in each portfolio increased over the 35-year period from a low of 58 securities per portfolio in 1931 to a high of 110 in 1965.

Month-by-month returns for the portfolios were computed from January 1931 to December 1965. Average portfolio returns and portfolio betas were computed for the 35-year period and for a variety of sub-periods.

The results for the complete period are shown in Table 9. The average monthly portfolio returns and beta values for the 10 portfolios are plotted in Figure 12.

The results indicate that over the complete 35-year period, average return increased by approximately 1.08 percent per month (13 percent per year) for a one-unit increase in beta. This is about three-quarters of the amount predicted by the capital asset pricing model. As Figure 12 shows, there appears to be little reason to question the linearity of the relationship over the 35-year period.

Black, Jensen, and Scholes also estimated the risk-return tradeoff for a number of subperiods.[25/] The slopes of the regression lines tend in most periods to understate the theoretical values, but are generally of the correct sign. Also, the subperiod relationships appear to be linear.

This paper provides substantial support for the hypothesis that realized returns are a linear function of systematic risk values. Also, it shows that the relationship is significantly positive over long periods of time.

Fama and MacBeth

Fama and MacBeth [6] have extended the Black-Jensen-Scholes tests to include two additional factors. The first is an average of the β_j^2 for all individual securities in portfolio p, designated $\hat{\beta}_p^2$. The second is a similar average of the residual standard deviations $(\hat{SE}_j)$ for all stocks in portfolio p, designated $\hat{SE}_p$. The first term tests for nonlinearities in the risk-return relationship, the second for the impact of residual variation.

The equation of the fitted line for the Fama-MacBeth study is given by

$$\overline{R}_p = \gamma_0 + \gamma_1 \hat{\beta}_p + \gamma_2 \hat{\beta}_p^2 + \gamma_3 \hat{SE}_p + \mu_p$$

$$(19)$$

where, according to the CAPM, we should expect γ_2 and γ_3 to have zero values.

The results of the Fama-MacBeth tests show that while estimated values of γ_2 and γ_3 are not equal to zero for each interval examined, their average values tend to be insignificantly different from zero. Fama and MacBeth also confirm the Black-Jensen-Scholes result that the realized values of γ_0 are not equal to $\overline{R}_f$, as predicted by the capital asset pricing model.

Summary of Test Results

We will briefly summarize the major results of the empirical tests.

1. The evidence shows a significant positive relstionship between realized returns and systematic risk. However, the relationship is not always as strong as predicted by the capital asset pricing model.

2. The relationship between risk and return appears to be linear. The studies give no evidence of significant curvature in the risk-return relationship.

3. Tests which attempt to discriminate between the effects of systematic and unsystematic risk do not yield definitive results. Both kinds of risk appear to be positively related to security returns. However, we believe that the relationship between return and unsystematic risk is at least partly spurious -- that is, partly reflecting statistical problems rather than the true nature of capital markets.

Obviously, we cannot claim that the capital asset pricing model is absolutely right. On the other hand, the empirical tests do support the view that beta is a useful risk measure and that investors in high beta stocks expect correspondingly high rates of return.

8. MEASUREMENT OF INVESTMENT PERFORMANCE

The basic concept underlying investment performance measurement follows directly from the risk-return theory. The return on managed portfolios, such as mutual funds, can be judged relative to the returns on unmanaged portfolios at the same degree of investment risk. If the return exceeds the standard, the portfolio manager has performed in a superior way, and vice versa.

Given this, it remains to select a set of "benchmark" portfolios against which managed portfolio performance can be evaluated. The Capital Asset Pricing Model (CAPM) provides a convenient and familiar set of portfolios; however, as discussed below, these are not the only portfolios which could be used. The CAPM benchmark portfolios are simply combinations of the riskless rate and market index. The return standard for a managed portfolio with average beta equal to β_p is equal to the risk-free rate plus β_p times the average realized risk premium on the market. The performance measure, α_p, is equal to the difference in the average returns between the portfolio and the standard; that is,

$$\alpha_p = \bar{R}_p - \bar{R}_F + \beta_p(\bar{R}_M - \bar{R}_F) \tag{20}$$

where $\bar{R}_p$, $\bar{R}_M$, and $\bar{R}_F$ are the average returns for the portfolio, market index, and riskless bond during the test period.

Estimated values of alphs ($\hat{\alpha}_p$) and beta ($\hat{\beta}_p$) are determined as discussed in Section 5 by regressing the portfolio risk premiums on the market risk premiums. Positive values of $\hat{\alpha}_p$ are indications of superior performance, negative values of inferior performance.

The interpretation of the estimated alpha, however, must take into consideration possible statistical measurement errors. As discussed in Section 5, the standard error of alpha (SE_α) is a measure of the extent of the possible measurement error. The larger the standard error, the less certain we can be that the measured alpha is a close approximation to the true value.[26/]

A measure of the degree of statistical significance of the estimated alpha value is given by the ratio of the estimated alpha to its standard error. The ratio, designated as t , is given by

$$t_\alpha = \frac{\hat{\alpha}_p}{SE_\alpha} \qquad (21)$$

The t_α gives a measure of the extent to which the true value of alphs can be considered to be different from zero. It measures the number of multiples of standard error that $\hat{\alpha}_p$ is away from zero. If the absolute value of t_α is large, then we have more confidence that the true value of alpha is different from zero. Absolute values of t_α in excess of 2.0 indicate a probability of less than about 2.5% that the true value of alpha could equal zero.

These methods of performance measurement were originally devised by Michael Jensen [10] [11] and have been widely used in many studies of investment performance, including that of the recent SEC Institutional Investor Study [20].

However, the tests of the capital asset pricing model summarized in Section 7 indicate that the average returns over time on securities and portfolios deviate systematically from the predictions of the model Though the observed average risk-return relationships seem to be linear, the tradeoff of risk for return is, in general, less than would be predicted

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from the CAPM. In short, the evidence suggests that the CAPM does not provide the best benchmarks for the average return-risk tradeoffs available in the market from naively selected portfolios.

These results do not prohibit our attempts to measure performance. They indicate that benchmark portfolios other than those prescribed by the CAPM would be more appropriate; but given such alternative naively selected portfolios, the analysis could proceed in exactly the same manner as described above. The work of Black, Jensen, and Scholes [1] shows the average return from naively selected portfolios, when plotted against risk, tends to lie along a straight line with slope somewhat less than implied by the CAPM. These "empirical risk return" lines would seem to be a natural alternative to the market line implied by the capital asset pricing model. Performance would then be measured relative to the empirical line, as opposed to the market line. A comparison of those two standards is illustrated in Figure 13. The market line performance measure (designated as α_1 in Figure 13) is equal to the vertical distance from the portfolio to the market line. The empirical line measure (designated α_2) is the vertical distance from the portfolio to the empirical line.

Since the market index ideally is composed of all assets, both the empirical and market lines would be expected to pass through the market index coordinates (point O in Figure 13). The intercepts on the return axis, however, are different. The market line intercept, by definition, is equal to the risk-free rate. The empirical line intercept equals the average return on a portfolio with "zero beta", designated R_Z.

The existence of long-run rates of return on the zero beta portfolio different from the riskless rate is a clear violation of the predictions of the CAPM. As of this time, there is no clear theoretical understanding as to the nature of this difference.

To summarize, empirically based performance standards would seem to be the natural alternative to those of the capital asset pricing model. This follows mainly because the empirical standards reflect the actual performance of naively selected portfolios. However, the design of appropriate empirical standards requires further research. In the interim, the familiar market line benchmarks can provide useful information regarding relative performance, but care must be exercised to avoid drawing fine distinctions among portfolio results.

9. CONCLUDING REMARKS

Our task is finally completed. We have presented a brief but hopefully comprehensive introduction to the foundations and tests of modern portfolio theory. Our aim was to provide the reader with a first view of the subject in hopes that his interest will be whetted for further study.[27/]

The major topics dealt with were the specification and measurement of security and portfolio risk, the development of a hypothesis for the relationship between expected return and risk, and the use of the resulting model to measure the performance of institutional investors. We have not provided a set of final answers to questions in these areas because none currently exist. The theory and empirical evidence are in a state of rapid evolution, and our knowledge has increased markedly in the recent past and will surely continue to do so in the future.
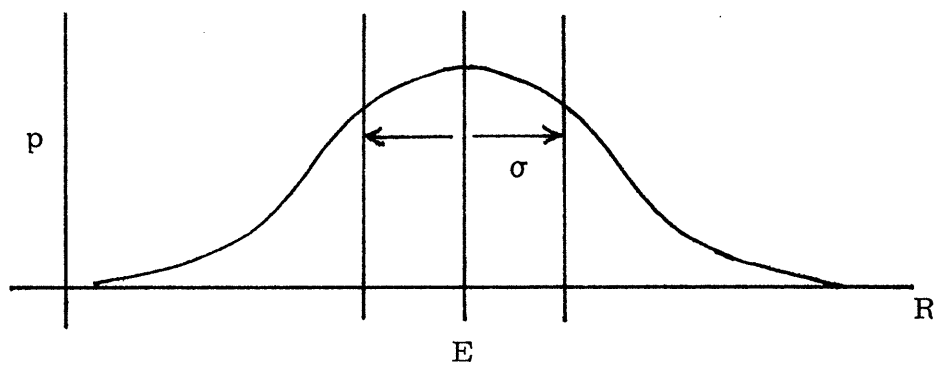
LIST OF FIGURES

Figure 1

POSSIBLE SHAPES FOR PROBABILITY DISTRIBUTIONS

1. Symmetric Probability Distribution

2. Probability Distribution Skewed to Left
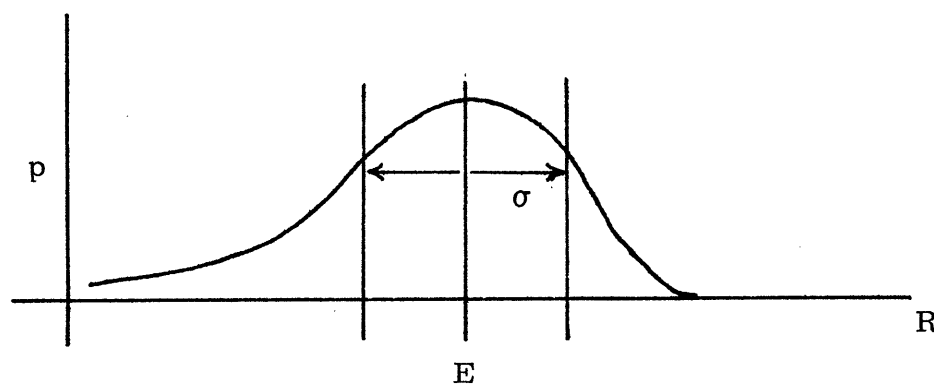
3. Probability Distribution Skewed to Right
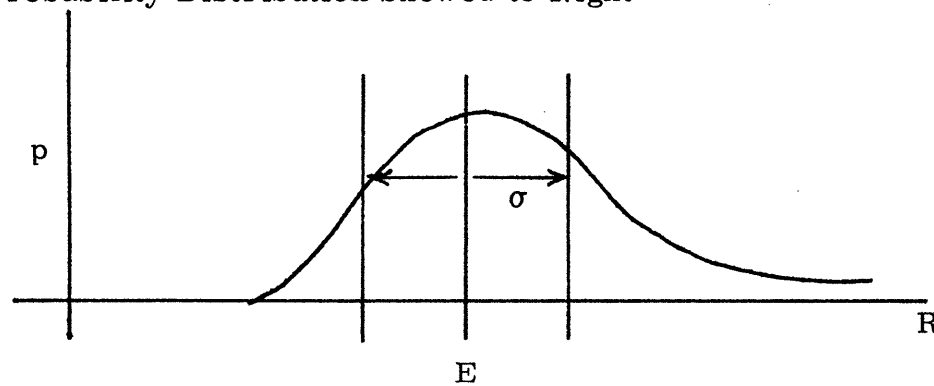
Figure 2

RATE OF RETURN DISTRIBUTION FOR A PORTFOLIO OF 100 SECURITIES

(EQUALLY WEIGHTED)

January 1945 - June 1970

	RANGE	FREQ.1...5...10...15...20...25...30...35...40...45...50.
1	-13.6210 -12.2685	1 *
2	-12.2685 -10.9160	2 **
3	-10.9160 -9.5635	2 **
4	-9.5635 -8.2110	3 8**
5	-8.2110 -6.8585	8 8******
6	-6.8585 -5.5060	9 *********
7	-5.5060 -4.1535	17 8****************
8	-4.1535 -2.8010	18 ******************
9	-2.8010 -1.4485	27 ***************************
10	-1.4485 -0.0960	28 8***************************
11	-0.0960 1.2565	30 ******************************
12	1.2565 2.6090	50 **
13	2.6090 3.9615	35 ***********************************
14	3.9615 5.3140	33 *********************************
15	5.3140 6.6665	18 8*****************
16	6.6665 8.0190	14 **************
17	8.0190 9.3715	4 ****
18	9.3715 10.7240	2 **
19	10.7240 12.0765	2 8*
20	12.0765 13.4290	3 ***

SCALING FACTOR = 1

Average Return = 0.91% per month
Standard Deviation = 4.45% per month
Number of Observations = 306

Figure 3

RATE OF RETURN DISTRIBUTION FOR NATIONAL DEPARTMENT STORES

January 1945 – June 1970

```
       RANGE            FREQ.1...5...10...15...20...25...30...35...40...45...50
 1  -32.3670  -29.4168    1  8
 2  -29.4168  -26.4666    0
 3  -26.4666  -23.5163    0
 4  -23.5163  -20.5661    1  8
 5  -20.5661  -17.6159    1  8
 6  -17.6159  -14.6657    3  8**
 7  -14.6657  -11.7155   13  8*************
 8  -11.7155   -8.7653   11  8***********
 9   -8.7653   -5.8151   39  8**************************************
10   -5.8151   -2.8649   47  8**********************************************
11   -2.8649    0.0853   45  8********************************************
12    0.0853    3.0355   34  8*********************************
13    3.0355    5.9857   28  8***************************
14    5.9857    8.9359   25  8************************
15    8.9359   11.8861   17  8****************
16   11.8861   14.8363   17  8****************
17   14.8363   17.7865    9  8********
18   17.7865   20.7366    8  8*******
19   20.7366   23.6868    5  8*****
20   23.6868   26.6370    2  8*

SCALING FACTOR = 1
```

Average Return = 0.81% per month
Standard Deviation = 9.02% per month
Number of Observations = 306

Figure 4

STANDARD DEVIATION VERSUS

NUMBER OF ISSUES IN PORTFOLIO
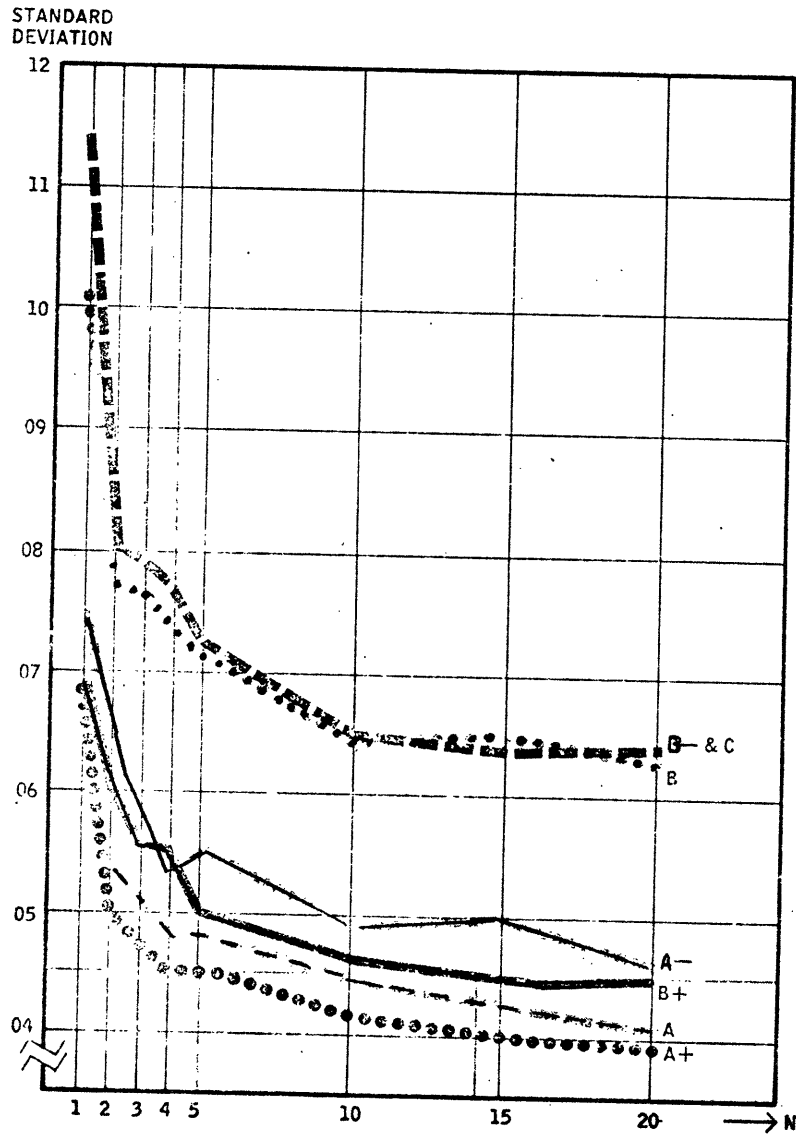
Source: Wagner and Lau [24], Exhibit 1.

Figure 5

CORRELATION VERSUS NUMBER OF

ISSUES IN PORTFOLIO
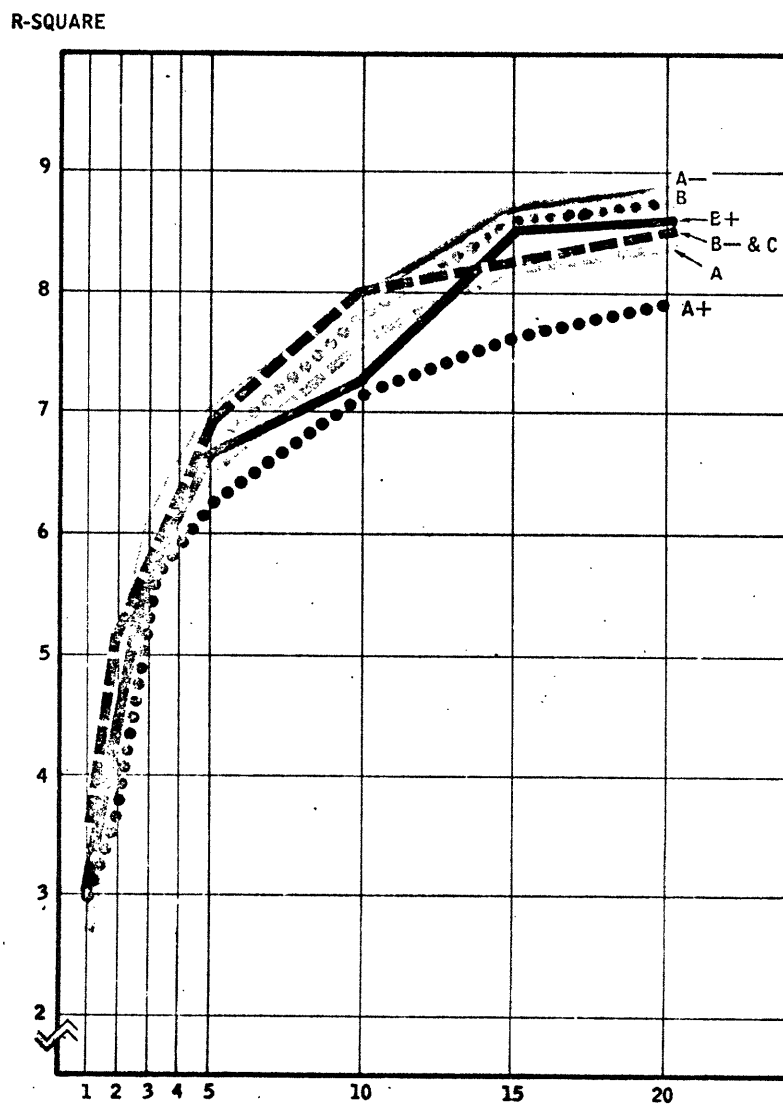
Source: Wagner and Lau [24],   Exhibit 2.

Figure 6

SYSTEMATIC AND UNSYSTEMATIC RISK
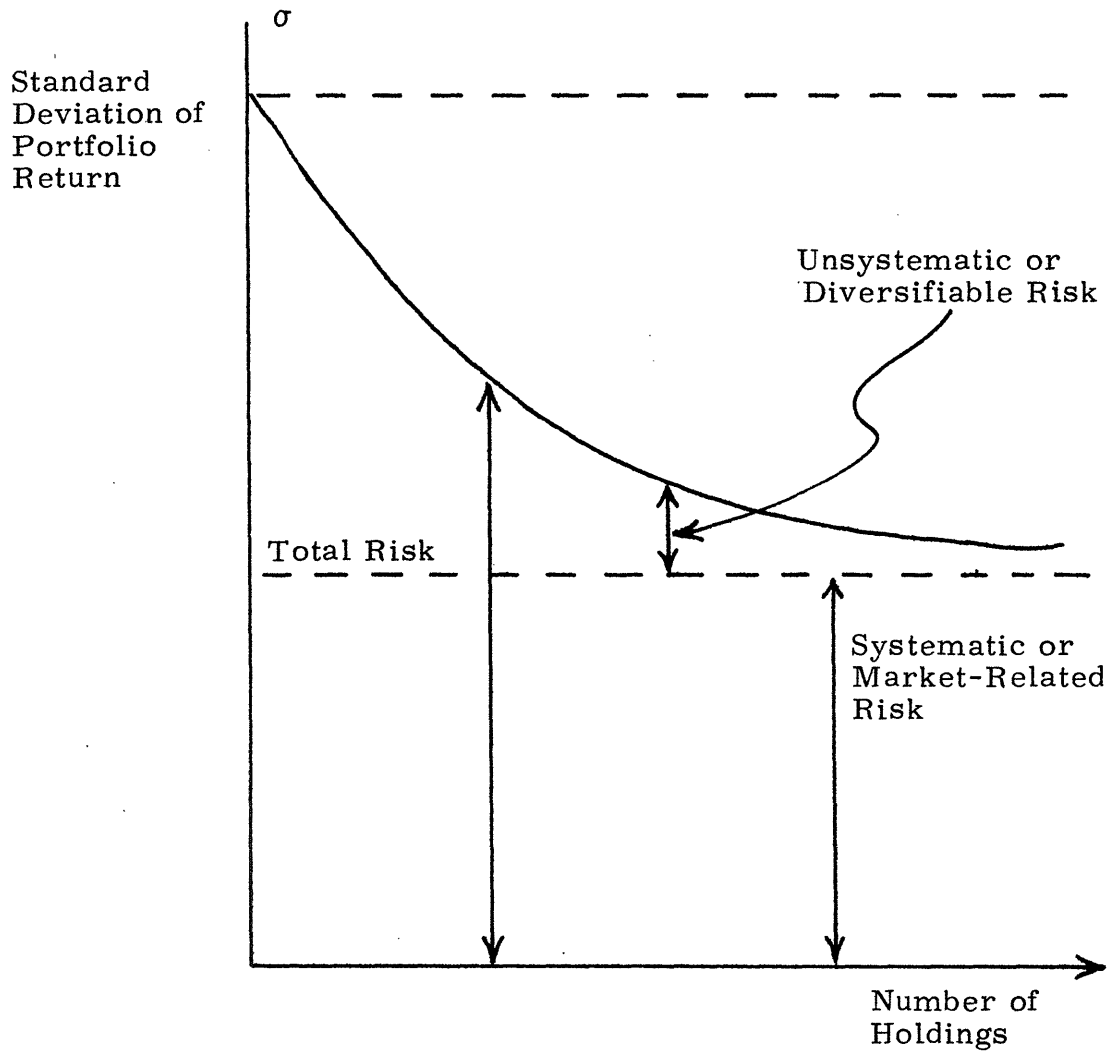
Figure 7

THE MARKET MODEL FOR SECURITY RETURNS
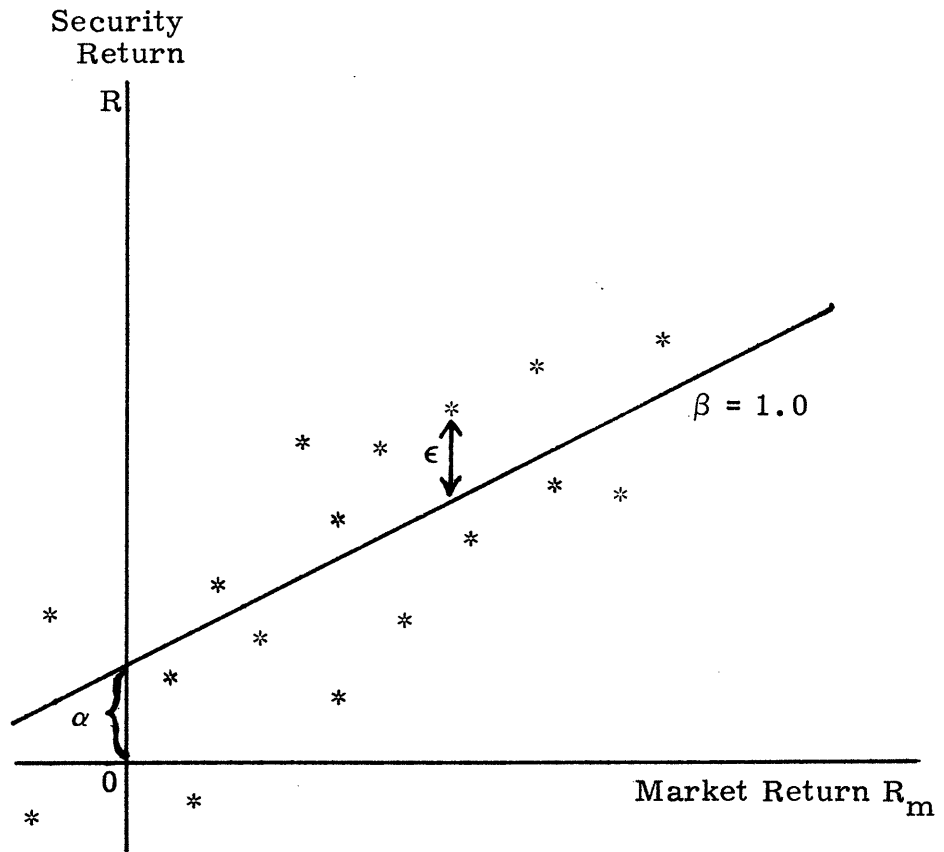
Beta (β), the market sensitivity index, is the slope of the line.

Alpha (α), the average of the residual returns, is the intercept of the line on the security axis.

Epsilon (ϵ), the residual returns, are the perpendicular distances of the points from the line

Figure 8

RETURNS ON NATIONAL DEPARTMENT STORES VERSUS NYSE INDEX

January 1945 – June 1970

NOBS	=	306
$\hat{\alpha}$	=	-0.05%
SE $_{\alpha}$	=	0.45%
$\hat{\beta}$	=	1.26
SE $_{\beta}$	=	0.12
$\hat{\sigma}_{\epsilon}$	=	7.73%
R^2	=	0.27
$\bar{r}$	=	0.81%
$\sigma(r)$	=	9.04%
$\bar{g}$	=	0.41%
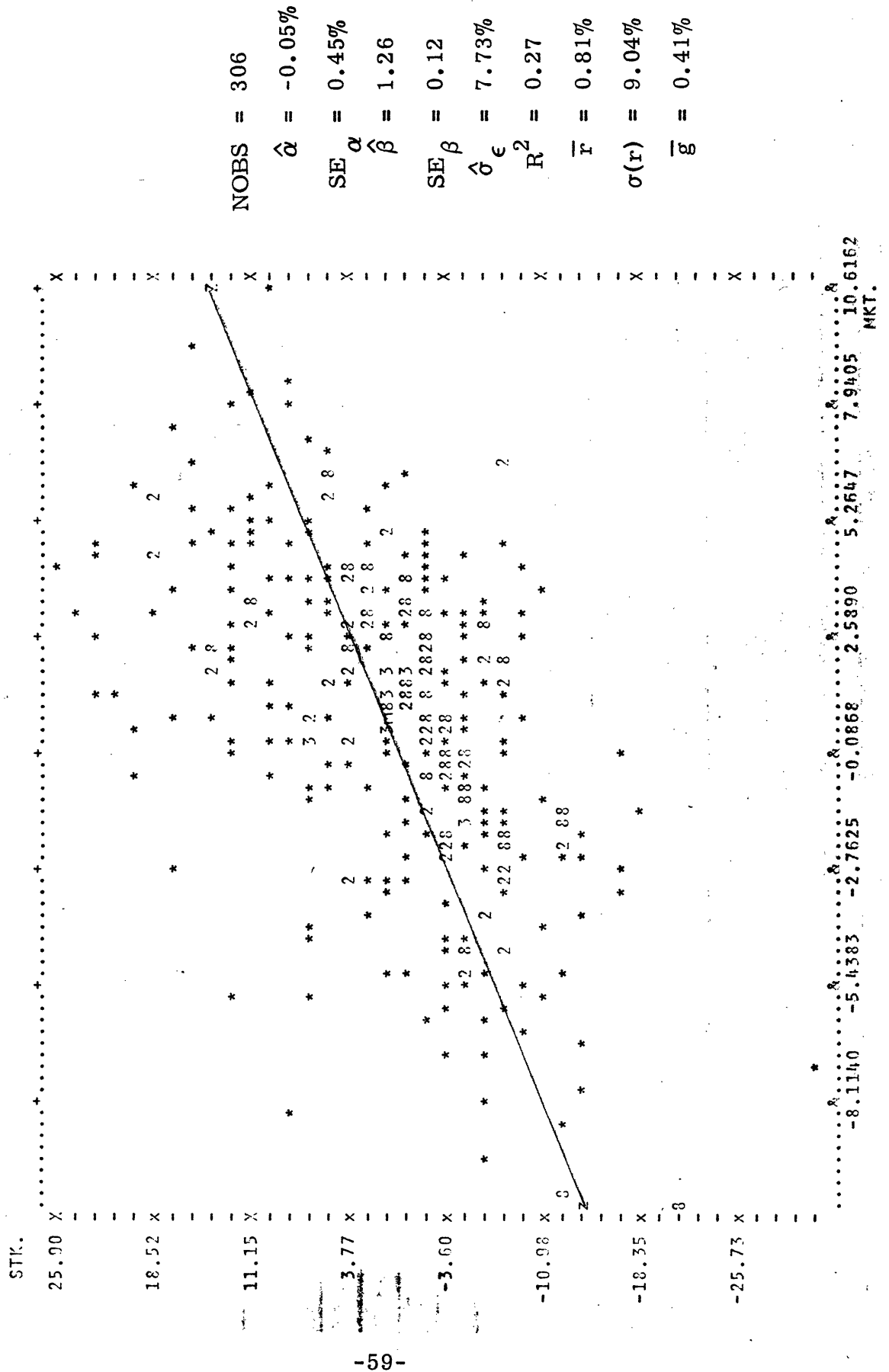
Figure 9

RETURNS ON 100 STOCK PORTFOLIO

VERSUS NYSE INDEX

January 1945 – June 1970

NOBS = 306

$\hat{\alpha}$ = 0.14%

SE α = 0.10%

$\hat{\beta}$ = 1.11
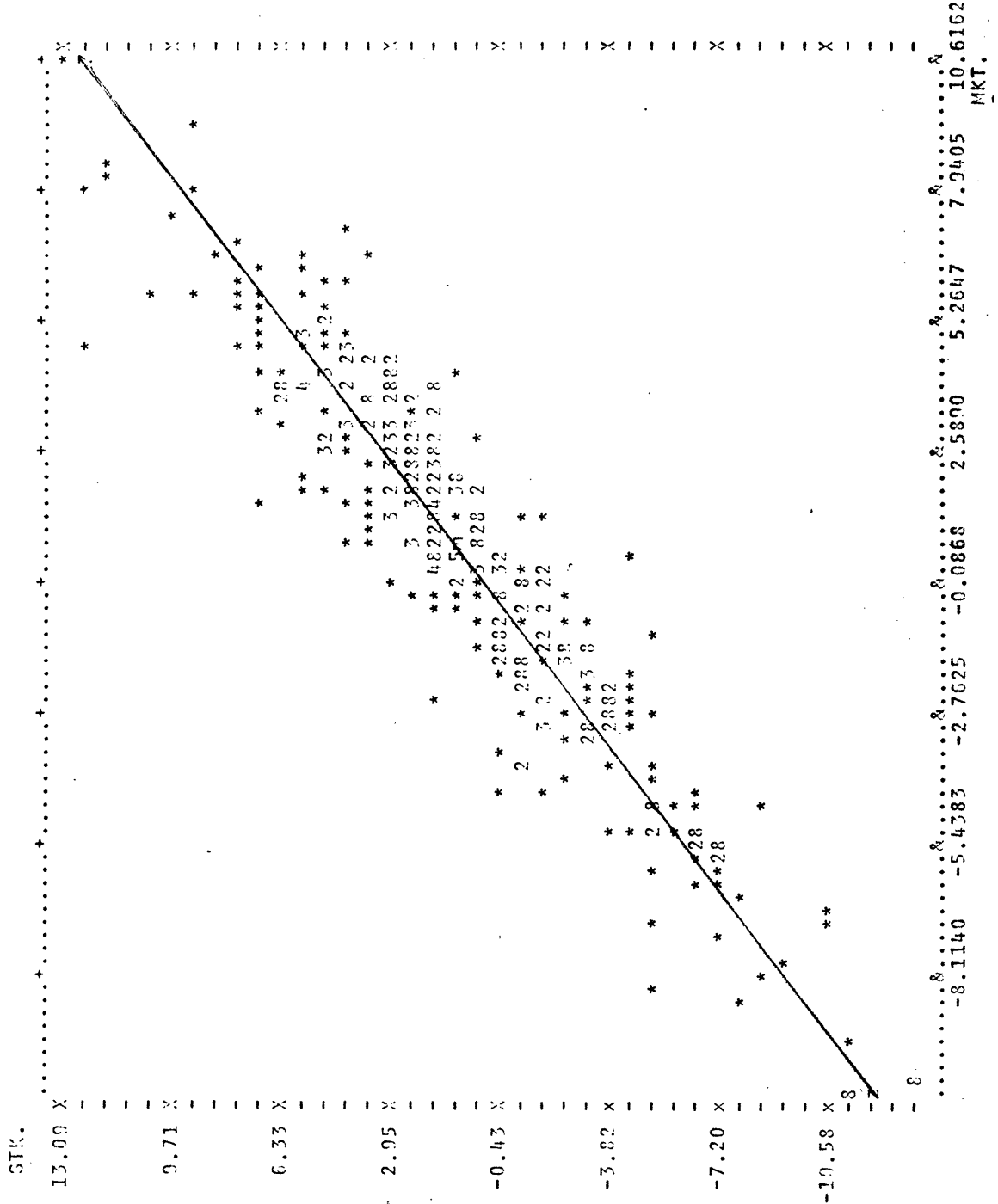
SE β = 0.03

$\hat{\sigma}_\epsilon$ = 1.64%

R^2 = 0.87

$\bar{r}$ = 0.91%

$\sigma(r)$ = 4.46%

$\bar{g}$ = 0.81%

Figure 10

INTERPERIOD BETA COMPARISON:

DAILY DATA FOR 90 MUTUAL FUNDS

Beta -- First Period (Jan '69 to May '70)
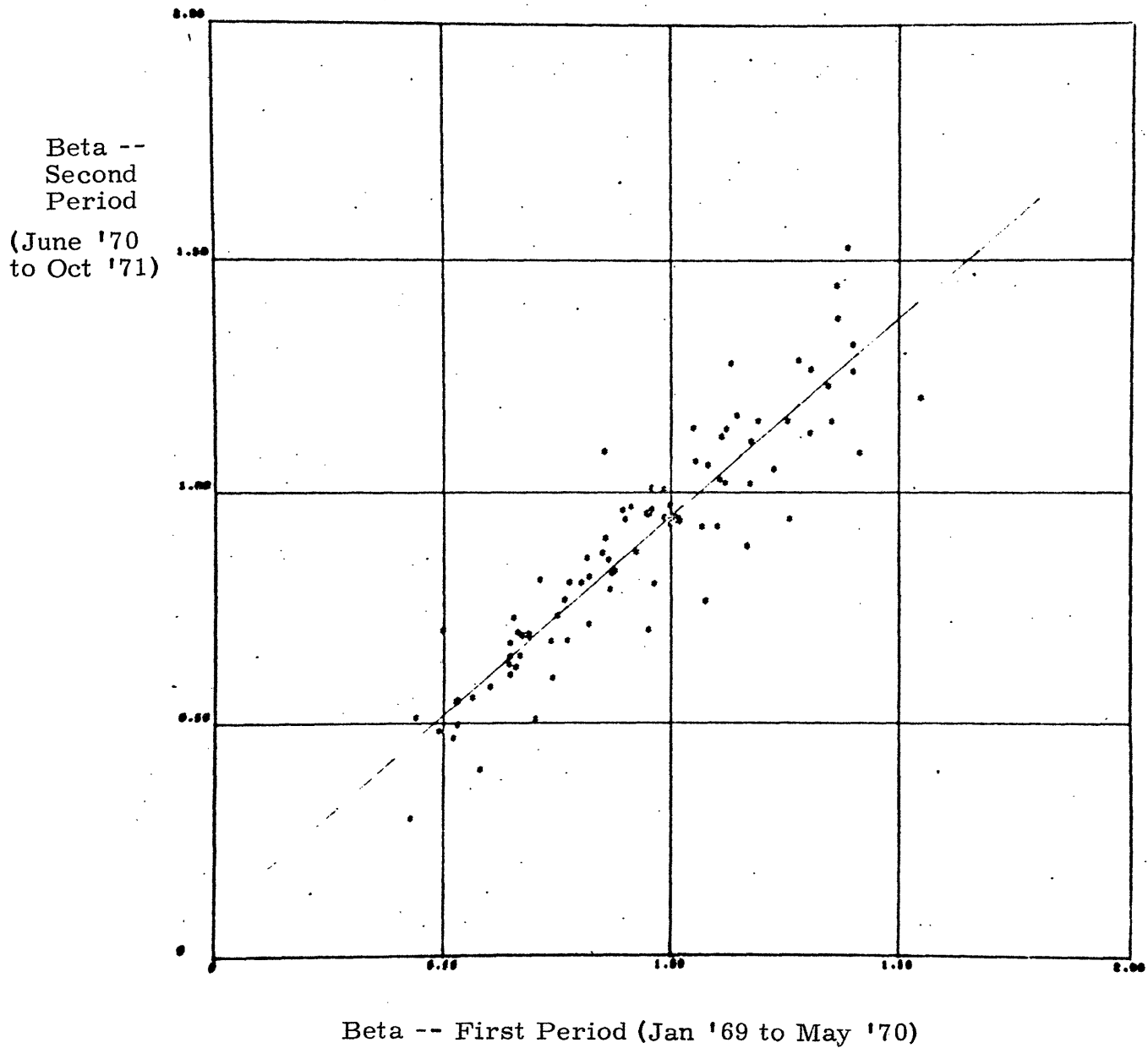
Source: Pogue and Conway [21]

Figure 11

RELATIONSHIP BETWEEN AVERAGE RETURN ($\overline{R}_j$)

AND SECURITY RISK (β_j)
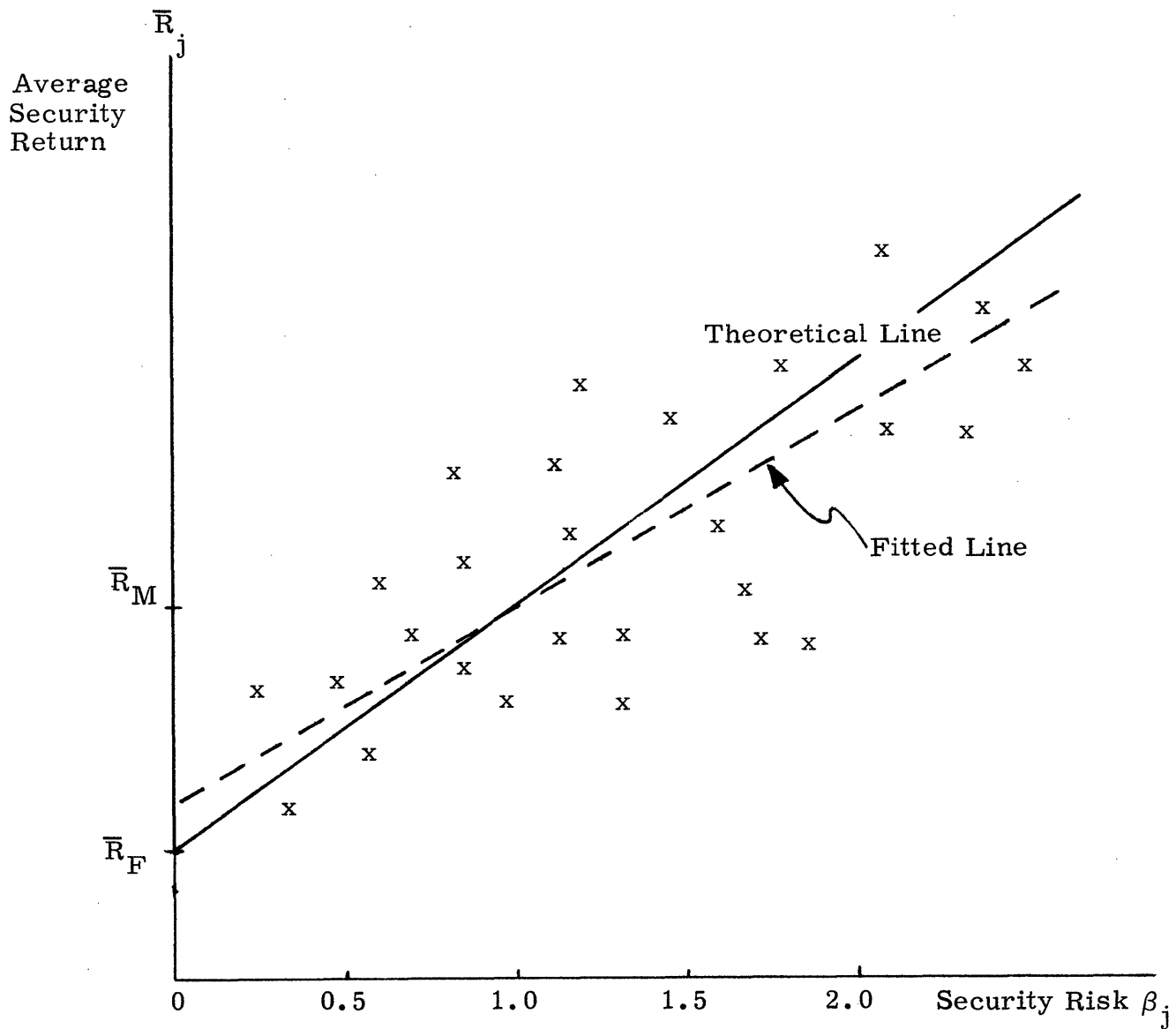
Figure 12

RESULTS OF BLACK, JENSEN AND SCHOLES STUDY

1931 - 1965
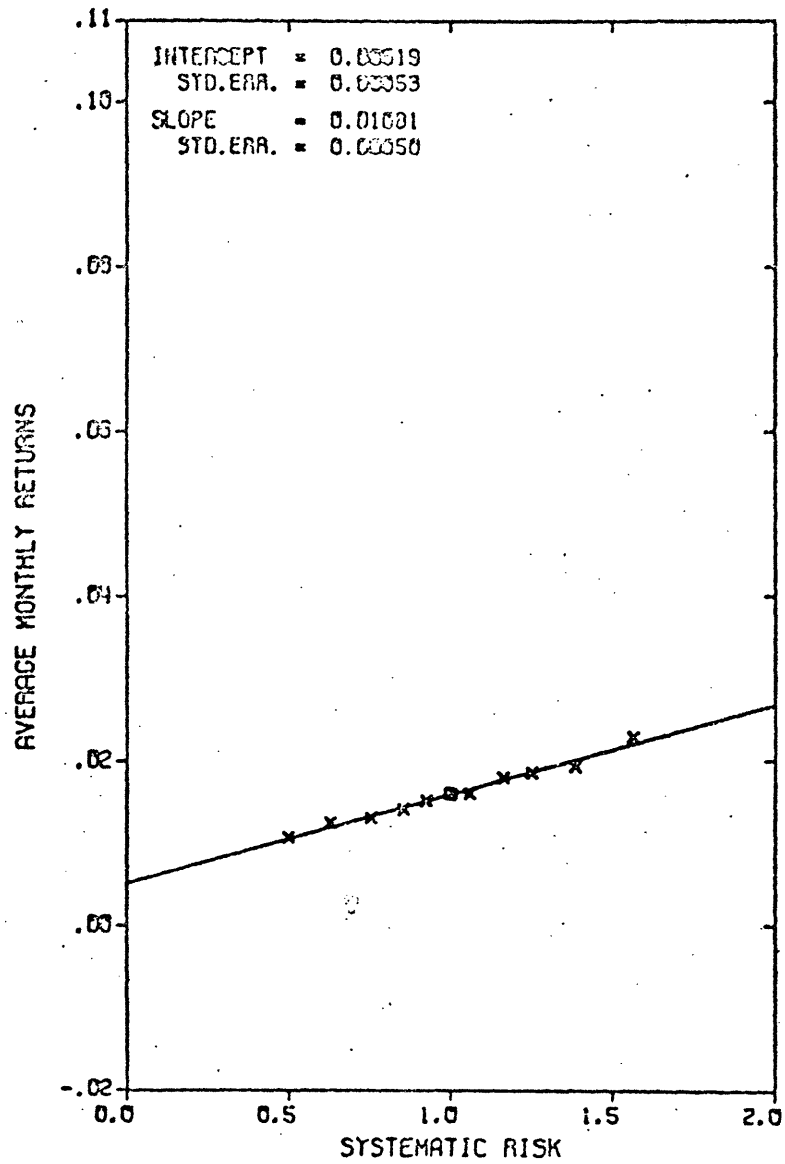
Average monthly returns versus systematic
risk for the 35-year period 1931-1965 for
ten portfolios and the market portfolio.

Source: Black, Jensen, and Scholes [1],
Figure 7.

Figure 13

MEASUREMENT OF INVESTMENT PERFORMANCE:

MARKET LINE VERSUS EMPIRICAL STANDARD
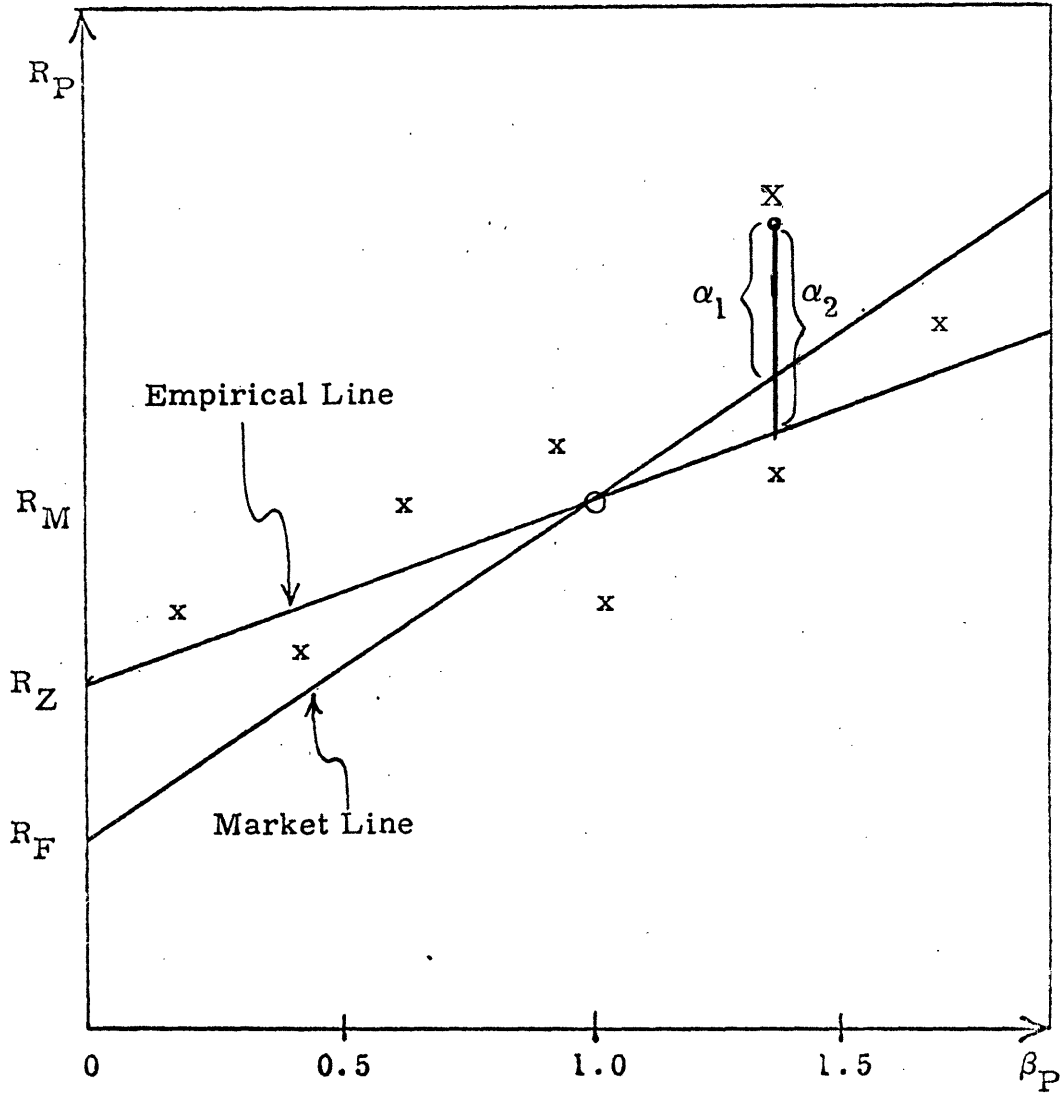
Symbols: R_M = Return on Market Index

R_Z = Return on Zero Beta Portfolio

R_F = Risk-Free Rate of Interest

X   = Investment Portfolios

O   = Market Index

Table 1

RISK VERSUS DIVERSIFICATION FOR
RANDOMLY SELECTED PORTFOLIOS
OF A+ QUALITY SECURITIES

June 1960 - May 1970

Number of Securities in Portfolio	Average Return (%/month)	Std. Deviation of Return (% / month)	Correlation with Market	
			R	R^2
1	0.88	7.0	0.54	0.29
2	0.69	5.0	0.63	0.40
3	0.74	4.8	0.75	0.56
4	0.65	4.6	0.77	0.59
5	0.71	4.6	0.79	0.62
10	0.68	4.2	0.85	0.72
15	0.69	4.0	0.88	0.77
20	0.67	3.9	0.89	0.80

Source: Wagner and Lau [24], Table C.

Table 2

STANDARD DEVIATIONS OF 20-STOCK PORTFOLIOS

AND PREDICTED LOWER LIMITS

June 1960 - May 1970

(1)	(2)	(3)	(4)
Stock Quality Group	Standard Deviation of 20-Stock Portfolios $\sigma \cdot \%/\text{mo}$	Average Beta Value for Quality Group $\overline{\beta}$	Lower Limit * $\overline{\beta} \cdot \sigma_m$ $\%/\text{mo}$
A+	3.94	0.74	3.51
A	4.17	0.80	3.80
A-	4.52	0.89	4.22
B+	4.45	0.87	4.13
B	6.27	1.24	5.89
B- & C	6.32	1.23	5.84

* σ_m = 4.75% per month.

Source: Wagner and Lau [24], page 6.

Table 3

REGRESSION STATISTICS FOR 30 RANDOMLY SELECTED SECURITIES

January 1945 - June 1970

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
SECURITY	NOBS	ALPH	SE.A	BETA	SE.B	SE.R	R**2	ARPJ	SD.R	CRPJ
1 CITY INVESTING CO	306.00	0.30	0.53	1.67	0.14	9.20	31.43	1.45	11.09	0.87
2 FOSTER WHEELER	306.00	-0.12	0.49	1.57	0.13	8.36	32.98	0.96	10.20	0.46
3 PENNSYLVANIA DIXIE C	306.00	-0.20	0.47	1.40	0.12	8.15	29.33	0.77	9.67	0.33
4 NATIONAL GYPSUM CO	306.00	-0.18	0.32	1.38	0.08	5.45	47.29	0.77	7.49	0.50
5 RADIO CORP OF AMERIC	306.00	0.02	0.38	1.35	0.10	6.60	37.02	0.95	8.30	0.62
6 FOX FILM CORP %A<	306.00	-0.04	0.53	1.31	0.14	9.15	22.35	0.87	10.36	0.38
7 INTERCONTINENTAL RUB	306.00	0.69	0.64	1.28	0.17	10.95	16.13	1.58	11.94	0.92
8 NATIONAL DEPARTMENT	306.00	-0.05	0.45	1.26	0.12	7.73	27.05	0.81	9.04	0.41
9 PHILLIPS JONES CORP	306.00	0.36	0.44	1.25	0.12	7.54	27.89	1.22	8.86	0.85
10 CHRYSLER CORP	306.00	-0.26	0.37	1.21	0.10	6.29	34.12	0.58	7.73	0.28
11 AMERICAN HIDE AND LE	306.00	0.55	0.66	1.16	0.17	11.36	12.78	1.35	12.14	0.67
12 ADAMS EXPRESS	306.00	0.11	0.23	1.16	0.06	3.93	54.87	0.91	5.84	0.75
13 CATERPILLAR TRACTOR	306.00	0.43	0.32	1.14	0.08	5.45	38.09	1.22	6.92	0.99
14 CONTINENTAL STEEL CO	306.00	0.21	0.36	1.12	0.10	6.22	31.31	0.99	7.50	0.72
15 MARLAND OIL CO	306.00	0.06	0.29	1.11	0.08	4.99	40.69	0.82	6.47	0.62
16 AIR REDUCTION CO. IN	306.00	-0.59	0.29	1.08	0.08	4.98	39.73	0.16	6.41	-0.05
17 NATIONAL AVIATION	306.00	0.22	0.39	1.04	0.10	6.71	25.15	0.94	7.74	0.65
18 NATOMAS CO	306.00	0.28	0.63	1.01	0.17	10.88	10.72	0.98	11.50	0.37
19 NYSE INDEX	306.00	0.0	0.0	1.00	0.0	0.0	0.0	0.69	3.73	0.62
20 AMERICAN SHIP BUILDI	306.00	0.31	0.52	0.99	0.14	9.01	14.53	0.99	9.73	0.54
21 JAMES TALCOTT	306.00	0.33	0.42	0.98	0.11	7.23	20.43	1.01	8.09	0.68
22 JEWEL TEA CO INC	306.00	0.21	0.32	0.95	0.08	5.42	30.14	0.87	6.47	0.66
23 INTERNATIONAL CARRIE	306.00	0.34	0.26	0.93	0.07	4.39	38.41	0.98	5.58	0.83
24 KEYSTONE STEEL & WIR	306.00	0.18	0.30	0.84	0.08	5.19	26.90	0.76	6.05	0.58
25 SWIFT & CO	306.00	-0.09	0.30	0.81	0.08	5.08	26.08	0.47	5.89	0.30
26 SOUTHERN CALIFORNIA	306.00	0.00	0.22	0.77	0.06	3.77	36.60	0.53	4.72	0.42
27 BAYUK CIGARS	306.00	-0.04	0.39	0.71	0.10	6.76	13.49	0.45	7.26	0.19
28 FIRST NATIONAL STORE	306.00	-0.08	0.31	0.67	0.08	5.33	18.01	0.38	5.88	0.21
29 NATIONAL LINEN SERVI	306.00	0.61	0.33	0.63	0.09	5.75	14.50	1.04	6.20	0.86
30 AMERICAN SNUFF	306.00	0.17	0.25	0.54	0.07	4.33	17.74	0.54	4.77	0.43
31 HOMESTAKE MINING CO	306.00	0.16	0.38	0.24	0.10	6.60	1.77	0.33	6.65	0.11
32 COMMERCIAL PAPER	306.00	0.0	0.0	0.0	0.0	0.0	0.0	0.28	0.17	0.28
...MEAN SEC. VALUES	306.00	0.13	0.39	1.05	0.10	6.76	27.25	0.86	7.88	0.54
...STANDARD DEVIATIONS	0.0	0.28	0.12	0.31	0.03	2.10	11.85	0.33	2.13	0.26

Table 4

REGRESSION STATISTICS FOR 49 MUTUAL FUNDS

January 1960 - December 1971

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
SECURITY	NOBS	ALPH	SE.A	BETA	SE.B	SE.R	R**2	ARPJ	SD.R	CRPJ
1MCDONNELL FUND INCOR	144.00	0.58	0.82	1.50	0.22	9.76	25.18	1.13	11.24	0.67
2VALUE LINE SPECIAL S	144.00	0.02	0.40	1.48	0.11	4.78	57.62	0.57	7.32	0.30
3KEYSTONE S-4	144.00	0.03	0.28	1.43	0.08	3.38	71.77	0.55	6.34	0.35
4CHASE FUND OF BOSTON	144.00	0.11	0.33	1.42	0.09	3.94	64.78	0.63	6.61	0.41
5EQUITY PROGRESS	144.00	-0.54	0.41	1.26	0.11	4.85	48.89	-0.08	6.77	-0.31
6FIDELITY TREND FUND	144.00	0.79	0.29	1.23	0.08	3.52	63.39	1.24	5.80	1.07
7FIDELITY CAPITAL FUN	144.00	0.41	0.24	1.20	0.06	2.81	72.17	0.85	5.31	0.71
8KEYSTONE K-2	144.00	0.08	0.22	1.17	0.06	2.63	73.90	0.51	5.13	0.38
9OPPENHEIMER FUND	144.00	0.67	0.31	1.16	0.08	3.66	58.86	1.10	5.69	0.94
10DELAWARE FUND	144.00	0.18	0.19	1.15	0.05	2.32	77.62	0.60	4.90	0.48
11KEYSTONE S-3	144.00	0.18	0.19	1.14	0.05	2.32	77.50	0.60	4.88	0.48
12PUTNAM GROWTH FUND	144.00	0.21	0.19	1.13	0.05	2.25	78.19	0.62	4.80	0.51
13SCUDDER SPECIAL FUND	144.00	0.39	0.28	1.12	0.07	3.33	61.93	0.80	5.37	0.66
14ENERGY FUND	144.00	0.06	0.18	1.10	0.05	2.18	78.39	0.46	4.67	0.35
15ONE WILLIAM STREET F	144.00	0.13	0.22	1.06	0.06	2.66	69.33	0.52	4.78	0.41
16THE DREYFUS FUND	144.00	0.17	0.14	1.04	0.04	1.69	84.40	0.55	4.26	0.46
17MASSACHUSETTS INVEST	144.00	0.15	0.16	1.03	0.04	1.96	79.65	0.52	4.34	0.43
18WINDSOR FUND	144.00	0.18	0.16	1.03	0.04	1.95	79.87	0.56	4.33	0.47
19AXE-HOUGTON STOCK FU	144.00	0.39	0.30	1.02	0.08	3.62	52.96	0.76	5.26	0.62
20S&P 500 STOCK INDEX	144.00	0.0	0.0	1.00	0.0	0.0	0.0	0.37	3.76	0.30
21T ROWE PRICE GROWTH	144.00	0.05	0.14	0.98	0.04	1.72	82.08	0.41	4.06	0.32
22MASSACHUSETTS INVEST	144.00	-0.02	0.14	0.97	0.04	1.72	82.07	0.34	4.04	0.26
23BULLOCK FUND	144.00	0.09	0.19	0.96	0.05	2.32	71.10	0.44	4.29	0.35
24KEYSTONE S-2	144.00	0.04	0.12	0.96	0.03	1.45	86.12	0.39	3.89	0.31
25EATON & HOWARD STOCK	144.00	-0.05	0.13	0.95	0.03	1.52	84.75	0.30	3.89	0.23
26THE COLONIAL FUND	144.00	0.06	0.19	0.95	0.05	2.27	71.24	0.41	4.23	0.32
27FIDELITY FUND	144.00	0.15	0.11	0.95	0.03	1.31	88.08	0.50	3.79	0.43
28INVESTMENT CO OF AME	144.00	0.26	0.20	0.95	0.05	2.40	68.79	0.61	4.29	0.51
29HAMILTON FUNDS-SERIE	144.00	-0.12	0.23	0.93	0.06	2.73	62.55	0.22	4.44	0.12
30AFFILIATED FUND	144.00	0.08	0.14	0.89	0.04	1.71	79.31	0.41	3.74	0.34
31KEYSTONE S-1	144.00	0.03	0.10	0.88	0.03	1.21	88.18	0.35	3.51	0.29
32AXE-HOUGHTON FUND B	144.00	0.01	0.20	0.86	0.05	2.44	63.68	0.32	4.03	0.24
33AMERICAN MUTUAL FUND	144.00	0.20	0.20	0.85	0.05	2.38	64.35	0.51	3.97	0.43
34PIONEER FUND	144.00	0.24	0.16	0.84	0.04	1.88	73.85	0.55	3.67	0.48
35CHEMICAL FUND	144.00	0.57	0.25	0.83	0.07	3.03	51.50	0.88	4.33	0.79
36STEIN ROW FARNHAM BA	144.00	0.06	0.10	0.79	0.03	1.21	86.05	0.35	3.22	0.30
37PURITAN FUND	144.00	0.19	0.15	0.78	0.04	1.79	72.89	0.48	3.43	0.42
38THE VALUE LINE INCOM	144.00	0.07	0.17	0.78	0.04	2.01	67.96	0.36	3.54	0.29
39THE GEORGE PUTNAM FU	144.00	0.07	0.10	0.77	0.03	1.18	85.75	0.35	3.12	0.30
40ANCHOR INCOME	144.00	-0.03	0.13	0.74	0.04	1.60	75.24	0.24	3.21	0.19
41LOOMIS-SAYLES MUTUAL	144.00	0.05	0.10	0.74	0.03	1.22	83.96	0.32	3.04	0.27
42WELLINGTON FUND	144.00	-0.12	0.13	0.72	0.03	1.54	75.60	0.14	3.11	0.09
43MASSACHUSETTS FUND	144.00	0.04	0.11	0.72	0.03	1.26	82.16	0.30	2.98	0.26
44NATIONWIDE SECURITIE	144.00	-0.32	0.15	0.67	0.04	1.78	66.45	-0.08	3.07	-0.12
45EATON & HOWARD BALAN	144.00	-0.07	0.12	0.62	0.03	1.46	71.62	0.16	2.74	0.12
46AMERICAN BUSINESS SH	144.00	0.12	0.09	0.53	0.02	1.10	76.96	0.31	2.28	0.29
47KEYSTONE K-1	144.00	0.01	0.11	0.53	0.03	1.32	69.59	0.21	2.39	0.18
48KEYSTONE-B-4	144.00	0.12	0.13	0.30	0.03	1.51	35.82	0.23	1.88	0.21
49KEYSTONE-B-2	144.00	0.05	0.10	0.16	0.03	1.16	22.03	0.11	1.31	0.10
50KEYSTONE B-1	144.00	-0.08	0.10	0.07	0.03	1.21	4.43	-0.06	1.23	-0.07
5130 DAY TREAS. BILLS	144.00	0.0	0.0	0.0	0.0	0.0	0.0	0.34	0.12	0.34
...MEAN SEC. VALUES	144.00	0.12	0.20	0.92	0.05	2.35	68.79	0.46	4.25	0.36
...STANDARD DEVIATIONS	0.0	0.23	0.12	0.30	0.03	1.42	17.39	0.28	1.65	0.24

Table 5

CORRELATION OF 52-WEEK BETA FORECASTS WITH
MEASURED VALUES FOR PORTFOLIOS OF N SECURITIES

1962 - 1970

Forecast for 52 Weeks Ended	Product Moment Correlations : N =				
	1	5	10	25	50
12/28/62	.385	.711	.803	.933	.988
12/27/63	.492	.806	.866	.931	.963
12/25/64	.430	.715	.825	.945	.970
12/24/65	.451	.730	.809	.936	.977
12/23/66	.548	.803	.869	.952	.974
12/22/67	.474	.759	.830	.900	.940
12/20/68	.455	.732	.857	.945	.977
12/19/69	.556	.844	.922	.965	.973
12/18/70	.551	.804	.888	.943	.985
Quadratic Mean	.486	.769	.853	.939	.972

Source: Robert A. Levy [13], Table 2.

Table 6

RESULTS OF JACOB'S STUDY

$$\overline{r}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \mu_j$$

Tests Based on 593 Securities

Period	Return Interval	Regression Results[a]			Theoretical Values	
		$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	$\gamma_0 = 0$	$\gamma_1 = \overline{R}_M - \overline{R}_F$
46-55	Monthly	0.80	0.30 (0.07)[b]	0.02	0	1.10
	Yearly	8.9	5.10 (0.53)	0.14	0	14.4
56-65	Monthly	0.70	0.30 (0.06)	0.03	0	0.8
	Yearly	6.7	6.7 (0.53)	0.21	0	10.8

(a) Coefficient units are: monthly data, percent per month;
annual data, percent per year.

(b) Standard error.

Source: Jacob [9], Table 3.

Table 7

RESULTS OF THE MILLER AND SCHOLES STUDY

$$\overline{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_2(\hat{SE}_j)^2 + \mu_j$$

Annual Rates of Return 1954-1963
Tests Based on 631 Securities

Regression Results[a]				Theoretical Values		
$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	R^2	γ_0	γ_1	γ_2
12.2 (0.7)[b]	7.1 (0.6)		0.19	2.8	8.5	0
16.3 (0.4)		39.3 (2.5)	0.28	2.8	8.5	0
12.7 (0.6)	4.2 (0.6)	31.0 (2.6)	0.33	2.8	8.5	0

(a) Units of Coefficients: percent per year.

(b) Standard error.

Source: Miller and Scholes [19],
Table 1.

Table 8

RESULTS OF FRIEND-BLUME STUDY

Returns from a yearly revision policy for
stocks classified by beta for various periods

Port-folio Number	Holding Period					
	1929-1969		1948-1969		1956-1969	
	Beta	Mean Return %	Beta	Mean Return %	Beta	Mean Return %
1	0.19	0.79	0.45	0.99	0.28	0.95
2	0.49	1.00	0.64	1.01	0.51	0.98
3	0.67	1.10	0.76	1.25	0.66	1.12
4	0.81	1.28	0.85	1.30	0.80	1.18
5	0.92	1.26	0.94	1.35	0.91	1.17
6	1.02	1.34	1.03	1.37	1.03	1.14
7	1.15	1.42	1.12	1.32	1.16	1.10
8	1.29	1.53	1.23	1.33	1.30	1.18
9	1.49	1.55	1.36	1.39	1.48	1.15
10	2.02	1.59	1.67	1.36	1.92	1.10

Monthly arithmetic mean returns

Source: Friend and Blume [8], Table 4.

Table 9

RESULTS OF BLACK-JENSEN-SCHOLES STUDY

$$R_p = \gamma_0 + \gamma_1 \hat{\beta}_p + \mu_p$$

1931 - 1965

Tests Based on 10 Portfolios
(Averaging 75 Stocks per Portfolio)

Regression Results[a]			Theoretical Values	
γ_0	γ_1	R^2	$\gamma_0 = \overline{R}_F$	$\gamma_1 = \overline{R}_M - \overline{R}_F$
0.519	1.08	0.90	0.16	1.42
$(0.05)^{[b]}$	(0.05)			

(a) Units of Coefficients: percent per month.
(b) Standard error.

Source: Black, Jensen, and Scholes [1],
Table 4 and Figure 7.

1.   Respectively Professor and Associate Professor of Finance,
Sloan School of Management, MIT. This paper was originally
prepared by us as Part I of "A Study of Investment Company
Incentive Fee Arrangements". The research was supported by
a grant from the Investment Company Institute, Washington, D.C.

2.   The general expression for the arithmetic mean return for a series
of N periods is given by

$$\bar{R} = \frac{1}{N} \sum_{t=1}^{N} R_t$$

where R_t is the investment return during period t, as measured
by Equation (1).

3.   The general expression for the geometric or compounded rate of
return over a series of N periods is given by

$$\bar{G} = [(1 + R_1)(1 + R_2) \ldots (1 + R_N)]^{1/N} - 1$$

$$= [\prod_{t=1}^{N} (1 + R_t)]^{1/N} - 1$$

The total return for the period is given by $(1 + \bar{G})^N - 1$.

4.   The relationship between the geometric and arithmetic mean
returns is such that the arithmetic mean is always equal to or
greater than the geometric average. The difference increases
as the dispersion of the returns increases. The following
equation developed by Young and Trent [25] shows the nature
of the difference.

$$\bar{R} = [\bar{G} + \sigma^2(R)]^{1/2}$$

where

$\bar{R}$ = the arithmetic average return

$\bar{G}$ = the geometric average return

$\sigma^2(R)$ = the variance of the series of returns

Rearranging,

$$\bar{G} = [\bar{R}^2 - \sigma^2(R)]^{1/2}$$

This relationship shows that $\bar{R}$ and $\bar{G}$ are equal only for an

asset with constant returns. For given $\bar{R}$, as $\sigma^2(R)$ increases,

the difference between the means will increase. Thus, $\bar{R}$

is a good approximation for $\bar{G}$ only when the variance of the R_t

returns is small.

5.   The transformation changes nothing of substance, since

$$\widetilde{M}_T = (1 + \widetilde{R}_p) M_0$$

$$= M_0 + M_0 \widetilde{R}_p$$

where

$\widetilde{M}_T$ = terminal portfolio value

$\widetilde{R}_p$ = portfolio return

Since $\widetilde{M}_T$ is a linear function of $\widetilde{R}_p$, any risk measures

developed for the portfolio return will apply equally to the

terminal market value.

6.   Risk measures based on below-average-value variation are

analytically difficult to deal with. H. Markowitz, in Chapter 9

of [18], develops a semivariance statistic which measures

variability below the mean and compares it with the more commonly used variance calculation.

7.   See for example M. E. Blume [2].

8.   This result follows easily for continuously compounded rates of return. The return for N periods, R_p, is simply the sum of the N one-period returns; that is,

$$R_p = R_1 + R_2 + \ldots + R_N$$

Now, if the one-period returns are independently and identically distributed with variance σ^2, then the variance of R_p will equal $N\sigma^2$, the standard deviation $\sqrt{N}\sigma$. The key assumption of independence of portfolio returns over time is realistic, since security returns appear to follow a random walk through time.

9.   Two securities with perfectly correlated return patterns will have a correlation coefficient of 1.0. Conversely, if the return patterns are perfectly negative correlated, the correlation coefficient will equal -1. Two securities with uncorrelated (i.e., statistically unrelated) returns will have a correlation coefficient of zero. The average correlation coefficient between securities on the NYSE is approximately 0.7.

10.   The reader may wish to verify that total risk (as measured by variance) really does equal the sum of systematic and unsystematic risks (also measured by variance). The relationship between the risk components is given by

$$\sigma^2 = \beta^2 \sigma_m^2 + \sigma_\epsilon^2 \qquad (5a)$$

This follows directly from Equation (5) and the assumption of independence of R_m and ϵ.

The R-squared term previously discussed is the ratio of systematic to total risk (both measured in terms of variance).

$$R\text{-squared} = \frac{\beta^2 \sigma_m^2}{\sigma^2}$$

Note also that the R-squared is the square of the correlation coefficient between security and market returns. Thus, the correlation coefficient is equal to the ratio of $\beta\sigma_m$ to σ.

11. Assuming the unsystematic returns (ϵ_j) of securities to be uncorrelated (reasonably true in practice), the unsystematic portfolio risk is given by

$$\sigma^2(\epsilon_p) = \sum_{j=1}^{N} X_j^2 \, \sigma^2(\epsilon_j)$$

where $^2(\epsilon_j)$ is the unsystematic risk for stock j. Assume the portfolio is made up of equal investment in each security and $\bar{\sigma}^2(\epsilon)$ is the average value of the $\sigma^2(\epsilon_j)$. Then, $X_j = 1/N$ and

$$\sigma^2(\epsilon_p) = \frac{1}{N^2} \cdot \left(N\bar{\sigma}^2(\epsilon) \right) = \frac{1}{N} \bar{\sigma}^2(\epsilon)$$

which obviously approaches zero as the number of issues in the portfolio increases.

12. That is, the risk premium for month t, r_t, is given by

$$r_t = R_t - RF_t$$

where

R_t = security return in month t

RF_t = risk-free return in month t

The conversion to risk premiums results in no substantive change but is consistent with theoretical developments discussed in Section 6.

13. The sample was picked to give the broadest possible range of security beta values. This was accomplished by ranking al NYSE securities with complete data from 1945-70 by their estimated beta values during this period. We then selected every twenty-fivth stock from the ordered list. The data was obtained from the University of Chicago CRSP (Center for Research in Security Prices) tape.

14. The commercial paper results in Table 3 are rates of return, not risk premiums. The risk premiums would equal zero by definition.

15. Correlation studies of this type tend to produce a conservative picture of the degree of beta coefficient stationarity. This results

from the fact that it is not possible to correlate the true beta values but only estimates which contain varying degrees of measurement error. Measurement error would reduce the correlation coefficient even though the underlying beta values were unchanged from period to period.

16. These results are consistent with those found by N. Mains in a later and more extensive study [16]. Mains correlated adjacent calendar year betas for a sample of 99 funds for the period 1960 through 1971. The betas were based on weekly returns. The average correlation coefficient for the 11 tests was 0.788, with individual values ranging from a low of 0.614 to a high of 0.871.

17. From this point on, "systematic risk" will be referred to simply as risk. "Total risk" will be referred to as total risk.

18. We use the term portfolio in a general sense, including the case where the investor holds only one security. Since portfolio return and (systematic) risk are simply weighted averages of security values, risk-return relationships which hold for securities must also be true for portfolios, and vice versa.

19. The material in Section 7 was also prepared as an appendix to testimony to be delivered before the Federal Communications Commission by S. C. Myers and G. A. Pogue.

20. $\hat{SE}_j$ is an estimate of the standard error of the residual term in Equation (16). Thus, it is the estimated value for $\sigma(\epsilon_j)$, the the unsystematic risk term defined in Equation (8). See column (6) of Tables 3 and 4 for typical values for securities and mutual funds.

21. For example, skewness in the distributions of stock returns can lead to spurious correlations between mean return and SE_j. See Miller and Scholes [19], pp. 66, 71.

22. Their expanded test equation is

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_2 (\hat{\beta}_j)^2$$

where, according to the capital asset pricing model, the expected value of γ_2 is zero.

23. Blume and Friend [3], p. 16.

24. Table 1, p. 15, of Blume and Friend [3] presents period-by-period regression results.

25. Figure 6 of Black, Jensen and Scholes [1] shows average monthly returns versus systematic risk for 17 non-overlapping two-year periods from 1932 to 1965.

26. See columns 2 and 3 of Table 4 for typical mutual fund $\hat{\alpha}$ and SE_α values.

27. There are a number of excellent references for further study of portfolio theory. Among these we would recommend books by Richard A. Brealey [4], Jack Clark Francis [7], and William F. Sharpe [23]. For a more technical survey of the theoretical and empirical literature, see Jensen [12].

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